NASA Contractor Report **181778**

Optimal Aeroassisted Coplanar Orbital Transfer Using an Energy Model

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FOREWORD

The work described in this report was performed by Information & Control Systems, Incorporated (ICS) under Contract Number NAS1-17493 for the National Aeronautics and Space Administration (NASA), Langley Research Center, Hampton, Virginia. The work was sponsored by the Aircraft Guidance and Control Branch of the Guidance and Control Division. Mr. Richard M. Hueschen was the NASA Technical Representative for the contract. Dr. Daniel D. Moerder of the Spacecraft Controls Branch also monitored the technical progress of the work. Dr. Nesim Halyo directed the technical effort at ICS.

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1. INTRODUCTION

The need for cost-effective and reliable space transportation systems has been accentuated with increasing world-wide competition to exploit space applications and the urgency of military payloads. The use of aerodynamic rather than propulsive forces to perform various types of orbit transfer can provide significant advantages in increased payload, reduced fuel weight and reduced launch activities. The economic superiority of aeroassisted orbital transfer over all-propulsive maneuvering has been demonstrated through various studies [1]-[13].

In the typical aeroassisted coplanar orbital transfer shown in Figure 1, the vehicle is initially in high earth orbit at radius, r_1, when a retro impulse, say ΔV_1, brings the vehicle into an elliptical orbit with perigee, say r_p. Whereas ideally, r_p would be the top of the atmosphere, for practical reasons it would be selected somewhere inside the atmosphere in a realistic aeroassisted orbital transfer maneuver.

The portion of the orbital transfer of particular interest in this study is the atmospheric flight portion which starts as the vehicle enters the atmosphere at a radial distance from the center of the earth, say r_a. By appropriate modulation of the lift and drag forces, the vehicle reduces its speed to a level corresponding to its final lower orbit radius, r_2. The atmospheric trajectory also determines the heating rate which the vehicle skin will be subjected to. Thus, it is necessary to select a trajectory which does not subject the vehicle to temperature levels higher than can reasonably be accommodated by the vehicle skin. At atmospheric exit, the vehicle enters an elliptical orbit with apogee at r_2. A circularizing impulse, ΔV_2, at the apogee puts the vehicle in the desired final orbit.

Aeroassisted coplanar orbit transfer has been studied in [4] where it is determined that a zero flight path angle at atmospheric exit results in the minimal recircularizing impulse,

ΔV_2, for single impulse maneuvers. Furthermore, the sensitivity of ΔV_2 to variations in the flight path angle is seen to be high.

This result stresses the importance of achieving the appropriate conditions at atmospheric exit. However, off-nominal atmospheric conditions can produce significant perturbations in the actual trajectory of the vehicle. Significant variations in the atmospheric density profile have been observed in shuttle flights. Such variations from the standard atmosphere can result in deviations from the nominal trajectory and perturb the atmospheric exit conditions. Since stochastic nonlinear optimization techniques are not currently practical, an alternative is to define an optimal control problem which can generate optimal trajectories from the current actual state to the desired final state at atmospheric exit, and thus adjust to off-nominal atmospheric conditions. This strategy requires fast and reliable algorithms for solving two-point-boundary-value problems (TPBVP), which requires further investigation beyond the current study. However, the ability to update the trajectory would result in small rather than large variations in the exit parameters.

In Section 2, a reduced order model of the equations of motion is developed. This second order model uses the vehicle's total energy as the independent variable instead of time. Reduction of the order has the advantage that it reduces the order of the TPBVP to be solved. Furthermore, it recognizes that coplanar orbit transfer is a problem of transferring the vehicle from one energy level to another. The choice of the control value is intuitively more a matter of how much energy the vehicle must lose rather than what time it is.

In Section 3, an optimal control problem to transfer the vehicle from an initial state to a specified final state is formulated. The necessary conditions are obtained in Section 4, and optimal trajectories are simulated in Section 5.

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2. EQUATIONS OF MOTION

In this section, we will develop a set of equations describing the motion of the aero-assisted orbital vehicle center of mass using total energy as the independent variable rather than time. These equations of motion will then be used in the formulation of an optimal control problem, where the control will be defined as a function of total energy. As a result, although the problem is solved in an open-loop mode, the control is defined in terms of energy, so that the control value is selected according to the total energy of the vehicle.

The equations of motion which will be used describe the dynamics of the c.g. of the vehicle in flight within the atmosphere with no propulsive forces used. The general equations of motion for this case are given by

$$\dot{\theta} = \frac{V \cos\gamma \cos\psi}{r \cos\phi} \tag{1}$$

$$\dot{\phi} = V \cos\gamma \sin\psi / r \tag{2}$$

$$\dot{r} = V \sin\gamma \tag{3}$$

$$\dot{V} = -\frac{D}{m} - g\sin\gamma + \omega^2 r \cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\sin\psi) \tag{4}$$

$$\dot{\gamma} = \frac{L\cos\mu}{mV} + \frac{V}{r} - \frac{g\cos\gamma}{V} + 2\omega\cos\phi\cos\psi$$
$$+ (\omega^2 r/V)\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\cos\psi) \tag{5}$$

$$\dot{\psi} = \frac{L \sin \mu}{mV \cos \gamma} - \frac{V}{r} \cos \gamma \cos \phi \tan \phi$$
$$+ 2\omega (\tan \gamma \cos \phi \sin \psi - \sin \phi) - (\omega^2 r / V \cos \gamma) \sin \phi \cos \phi \cos \psi \qquad (6)$$

where θ is the longitude, ϕ is the latitude, r is the distance from the center of the earth to the vehicle center of gravity, V is the velocity with respect to the earth, γ is the local flight path angle, ψ is the track angle, μ is the bank angle, ω is the angular velocity of the earth, m is the mass of the vehicle, D is the drag force and L is the lift force acting on the vehicle.

For the case of coplanar orbital transfer considered here, the general equations of motion can be simplified significantly. First, we assume that a lateral regulator control system is maintaining the vehicle's lateral variables near zero by accommodating perturbations due to atmospheric and other effects. This implies the ability to bank the vehicle by small amounts to correct for small perturbations in the lateral variables. Taking the initial heading as zero, the latitude, ϕ, remains constant and can also be taken as zero.

The remaining equations of motion are (1), (3), (4) and (5). With the latitude/longitude directions as defined above, the motion of the vehicle is along zero-latitude and the position of the vehicle is determined by its longitude and altitude. In the current study, the longitude of the vehicle as a function of time is not of interest, and can be eliminated from the model since it does not impact the motion.

Rewriting the remaining equations of motion, we obtain

$$\dot{r} = V \sin \gamma \qquad (7)$$

$$\dot{V} = -\frac{D}{m} - \frac{\bar{\mu} \sin \gamma}{r^2} \qquad (8)$$

$$\dot{\gamma} = \frac{L}{mV} + \left(\frac{V}{r} - \frac{\bar{\mu}}{r^2 V} \right) \cos \gamma \qquad (9)$$

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where the angular rotation of the earth has been neglected and a Newtonian gravitational model is used, i.e.,

$$g = \frac{\bar{\mu}}{r^2} \tag{10}$$

in which the gravitational acceleration, g, is expressed in terms of the inverse square law with $\bar{\mu}$ being the product of the gravitational constant and the mass of the earth.

Furthermore, we assume a parabolic drag polar form between the coefficients of drag and lift:

$$C_D = C_{D_o} + K C_L^2 \tag{11}$$

$$\frac{L}{m} = \frac{1}{2} C_L \rho \bar{S} V^2 \tag{12}$$

$$\bar{S} = \frac{S}{m} \quad , \tag{13}$$

$$\frac{D}{m} = \frac{1}{2} C_D \rho \bar{S} V^2 \tag{14}$$

where ρ is the atmospheric density, and S is the vehicle's effective surface area for aerodynamic forces, C_{D_o} is the zero-lift or minimum coefficient of drag and K is a coefficient principally depending on the vehicle configuration shape.

Energy as Independent Variable.

In the model described above, the motion variables r, V and γ are the dependent variables; while the time, t, is the independent variable. However, it is possible to use a different independent variable, if some advantages accrue from a different choice. Of course, the independent variable cannot be selected arbitrarily. Knowledge of the independent variable must uniquely determine the value of each of the dependent variables. For

example, altitude cannot be used as the independent variable, since in a standard maneuver, the vehicle crosses the same altitude twice, once when dipping into the atmosphere at a high velocity and again on its way up to the new orbit at a lower velocity. Thus, knowing the altitude does not uniquely determine the velocity of the vehicle.

In the following, we will develop a new model of the motion of the vehicle using its total energy, potential plus kinetic, as the independent variable. First note that energy is allowable as an independent variable because it is monotonic. Since the aeroassisted orbital transfer vehicle (AOTV) will use no propulsive forces during its maneuver inside the atmosphere, the vehicle's total energy monotonically decreases with time due to atmospheric drag. Thus, energy is a one-to-one and invertible function of time; so that the vehicle has a given energy level only once during the maneuver. Knowing the vehicle's energy uniquely determines the value of the dependent variables.

The advantages of using energy as the independent variable are two-fold. First, it provides a technique of model order reduction analytically without any approximation. In optimal control problems, reducing the model order by one reduces the order of the two-point-boundary-value problem which must be solved by two, since the co-state equations are also reduced by one.

The second advantage is that coplanar orbit transfer intuitively is a problem of energy management; i.e., it is a problem of going from a high energy level to a lower energy level by losing some energy to the atmosphere without overheating. Thus, deciding what control value to use is intuitively more a question of how much energy the vehicle has rather than what time it is.

Define the vehicle's total energy, potential plus kinetic, say E as

$$E = \frac{1}{2}mV^2 - mgr = \frac{1}{2}mV^2 - \frac{m\bar{\mu}}{r} \tag{15}$$

$$\frac{E}{m} = \frac{V^2}{2} - \frac{\bar{\mu}}{r} \tag{16}$$

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Differentiating E with respect to time and combining with the equations of motion, we obtain

$$\dot{E} - V\,D = -V\,D \quad . \qquad (17)$$

As expected, the time rate of change of the vehicle's energy is work done by the aerodynamic forces acting on it; i.e., the force multiplied by the velocity in the direction of the force.

When performing an aeroassisted coplanar orbit transfer, usually the initial energy is higher than the final energy. Thus, during the typical maneuver, E progresses in the negative direction, whereas the standard variational equations for optimal control problems assume the independent variable progressing in the positive direction. To use the standard equations, we simply make the change of variables

$$e = -E/m \quad , \quad \dot{e} = -\dot{E}/m = V\,D/m \qquad (18)$$

Now, if the vehicle's energy and its speed are both known, then its altitude is uniquely determined from the potential energy. Thus, it can be found that

$$\frac{\mu}{r} = e + \frac{V^2}{2} \quad . \qquad (19)$$

Using the chain rule and (19), the equations of motion (7), (8), (9) can be expressed in term of the speed and flight path angle viewed as functions of energy. After some manipulation, we obtain

$$V' = -\frac{1}{V} - \frac{(V^2 + 2e)^2 \sin\gamma}{2\bar{\mu}\,\rho\,\bar{S}\,V^3\,C_D} \qquad (20)$$

$$\gamma' = \frac{1}{V^2}\left(\frac{C_L}{C_D}\right) + \frac{(V^4 - 4\,e^2)\cos\gamma}{2\bar{\mu}\,\rho\,\bar{S}\,V^4\,C_D} \qquad (21)$$

where the prime " $'$ " denotes the derivative with respect to e; i.e.,

$$V' = \frac{dV}{de} \quad , \tag{22}$$

$$\gamma' = \frac{d\gamma}{de} \quad . \tag{23}$$

It is seen that the second order model in (20), (21) is sufficient to describe the motion of the vehicle. Note that this reduction in the order of the model has been obtained without any approximation, by simply using energy as the independent variable.

The atmospheric density, ρ, is a function of altitude. For the purpose of the optimization study, the usual exponential form will be used to describe the atmospheric density profile with altitude. Thus,

$$\rho(h) = \rho_o \, e^{-h/\beta} \quad , \tag{24}$$

$$h = r - r_E \quad , \tag{25}$$

where r_E is the average earth radius and $\bar{\beta}$ the scale height of the exponential atmosphere.

Since the radial distance, r, is no longer a state variable, the altitude must be expressed in terms of the model variables.

$$h = \frac{2\bar{\mu}}{V^2 + 2e} - r_E \tag{26}$$

Thus, the two differential equations (20), (21), the quadratic drag polar (12) and the atmospheric density (24), (26) form a reduced order model describing the motion of the vehicle center of gravity.

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3. THE OPTIMIZATION PROBLEM

The maneuver considered is to transfer the AOTV from high earth orbit to low earth orbit by grazing through the atmosphere to lose energy. While in high orbit at a radial distance of r_1, a tangential retro impulse, ΔV_1, is applied, which puts the vehicle into an elliptical orbit with perigee at r_p. While ideally, r_p would be at the top of the atmosphere, say r_a, realism considerations require a lower altitude which would ensure that the vehicle dips into the atmosphere sufficiently to loose the required energy within a reasonable period of time. The vehicle then exits the atmosphere at a lower speed of V_ℓ and flight path angle γ_ℓ starting an elliptical orbit with apogee, r_2. When the vehicle reaches r_2, a tangential circularizing impulse, ΔV_2, brings it to the desired circular orbit.

It is well-known that in comparison to the all-propulsive orbit transfer, the aeroassisted maneuver requires significantly less fuel to achieve the same orbital transfer [4]. Also note that although ΔV_1 is the larger impulse, the variability of this impulse is quite small; i.e., the variation in ΔV_1 which achieves an orbit with a perigee anywhere within the atmosphere is quite small. On the other hand, the variation in ΔV_2 with variations in the exit flight path angle and velocity is significant. Furthermore, the minimum ΔV_2 impulse occurs when the exit flight path angle, γ_ℓ, is zero, assuming a single impulse from atmospheric exit to the low earth orbit [4]. Therefore, it is highly desirable to achieve the needed exit conditions to reduce the variability in the amount of fuel necessary to ensure the maneuver.

The atmospheric portion of the orbital transfer is the part of interest in this study. An important consideration during the atmospheric maneuver is to maintain the vehicle's skin temperature at acceptable levels. This is directly related to the heating rate produced by the atmospheric conditions. On the other hand, off-nominal atmospheric conditions can produce significant perturbations. Although nonlinear stochastic optimization is the most

direct approach to the treatment of random atmospheric phenomena, it is beyond the available resources of the current study. Instead, we will define a deterministic optimal control problem which, if necessary, can be solved on-line to adjust to changing atmospheric conditions, so that the guidance trajectory may change with varying atmospheres but still achieve the desired exit conditions. Alternatively, the trajectories may be computed off-line and stored, although stringent storage requirements would be placed on the on-board computer.

To accommodate the various objectives and constraints discussed, an optimal control problem may be posed as follows. Since the exit conditions largely determine the fuel requirements, the final flight path angle and the speed are considered to be fixed by the low earth orbit radius, r_2. On the other hand, the initial conditions are determined by the high earth orbit radius, r_1. The high entry speed, V_h, and flight path angle, γ_h, are also considered to be fixed by the particular maneuver. The main objective during the atmospheric maneuver, beyond achieving the atmospheric exit conditions, is to avoid overheating the vehicle skin. To a lesser extent, it may be of interest to avoid excessive shear stress on the skin of the vehicle. To achieve these objectives, the cost function is selected as a linear combination of the square of the heating rate and the drag force acting on the skin integrated over the entire atmospheric maneuver.

The control is the coefficient of lift, C_L. The coefficient of drag, C_D, is determined by C_L through (12). Both positive and negative values are allowed for C_L. Whether a negative value of lift is obtained by a negative pitch angle or by a positive pitch angle with the vehicle flying upside down would depend on the vehicle and implementation considerations.

Now, the vehicle speed at atmospheric entry determines the initial energy per mass, say e_1. Alternatively, the high earth orbit energy diminished by the impulsive energy of ΔV_1 also determines the energy at atmospheric entry. Similarly, let the energy at atmospheric exit correspond to e_2. Recall that e is the negative of the energy per unit

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mass, so that $e_1 < e_2$. Thus,

$$V(e_1) = V_h \quad , \quad \gamma(e_1) = \gamma_h \tag{27}$$

$$V(e_2) = V_\ell \quad , \quad \gamma(e_2) = \gamma_\ell \tag{28}$$

The heating rate is computed using the expression

$$\dot{Q} = A \rho^{\frac{1}{2}} V^3 \quad , \quad A = 3.08 \times 10^{-4} \quad . \tag{29}$$

A number of different expressions are available for the heating rate which in general, would depend on further atmospheric variables. Since only the trends are of interest here, (29) is considered sufficient.

We will use a cost function which is a linear combination of the integrated heating rate squared and the drag; i.e.,

$$J = \int_{t_1}^{t_2} \left[c_1 \dot{Q}^2 + c_2 D/m \right] dt \quad , \tag{30}$$

where t_1 and t_2 are the initial and final times, respectively.

Since the independent variable is e, J must be expressed in terms of e. From (18),

$$\frac{dt}{de} = \frac{1}{V \, D/m} \tag{31}$$

Using the chain rule and manipulating,

$$J = \int_{e_1}^{e_2} \left[\bar{c}_1 \frac{V^3}{C_D} + \bar{c}_2 \frac{1}{V} \right] de = \int_{e_1}^{e_2} g(V, \gamma, C_L) \, de \quad . \tag{32}$$

where

$$\bar{c}_1 = \frac{2 \, A^2 \, c_1}{\bar{S}} \quad , \quad \bar{c}_2 = c_2 \quad . \tag{33}$$

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It is interesting that while the final time, t_2, in the original problem would be free, in the energy model the final energy is fixed. Since the initial and final (or desired) orbits are known, the amount of energy which must be expended in the atmosphere is also known, although the duration of the atmospheric maneuver is not fixed, and is a part of the optimization.

Thus, the problem of obtaining aeroassisted coplanar orbit transfer trajectories can be posed as the optimal control problem of minimizing the cost function, J, in (32) while satisfying the constraints (20), (21), (27) and (28).

4. NECESSARY CONDITIONS

Using standard variational calculus texts [14], [15], it is possible to determine conditions which are necessary for optimality. Following this approach, the Hamiltonian, $\mathcal{H}$, is

$$\mathcal{H}(V, \gamma, C_L, p_V, p_\gamma, e) = g(V, \gamma, C_L) + p_V\, V' + p_\gamma\, \gamma' \quad , \tag{34}$$

where p is the co-state vector defined by

$$p_V' = \frac{dp_V}{de} = -\frac{\partial \mathcal{H}}{\partial V}(V, \gamma, C_L, p_V, p_\gamma, e) \quad . \tag{35}$$

Substituting (32), (20) and (21) into (34), the Hamiltonian for the problem is found to be

$$\mathcal{H}(V, \gamma, C_L, p_V, p_\gamma, e) = h_1(V, \gamma, p_V, p_\gamma, e)\frac{1}{C_D}$$
$$+ h_2(V, \gamma, p_V, p_\gamma)\left(\frac{C_L}{C_D}\right) + h_3(V, \gamma, p_V, p_\gamma) \tag{36}$$

$$h_1(V, \gamma, p_V, p_\gamma, e) = \bar{c}_1 V^3 - p_V \sin\gamma\frac{[V^2 + 2e]^2}{2\bar{\mu}\,\rho\,\bar{S}\,V^3} + p_\gamma \cos\gamma\frac{[V^4 - 4e^2]}{2\bar{\mu}\,\rho\,\bar{S}\,V^4} \tag{37}$$

$$h_2(V, \gamma, p_V, p_\gamma) = \frac{p_\gamma}{V} \tag{38}$$

$$h_3(V, \gamma, p_V, p_\gamma) = \frac{\bar{c}_2 - p_V}{V} \tag{39}$$

The differential equations for the co-state vector can be obtained from (35)-(39).

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$$p'_V = -3\bar{c}_1\frac{V^2}{C_D} + p_V\left\{-\frac{1}{V^2} + 2\left(V^{1/2} + \frac{2e}{V^{3/2}}\right)\left(\frac{1}{2}V^{-1/2} - \frac{3e}{V^{5/2}}\right)\frac{\sin\gamma}{2\bar{\mu}\rho\bar{S}C_D} - \frac{2\sin\gamma}{\bar{\beta}\rho\bar{S}V^2C_D}\right\}$$

$$+p_\gamma\left\{\left[\frac{2}{V^3}\left(\frac{C_L}{C_D}\right) - \frac{16e^2}{V^5}\right]\frac{\cos\gamma}{2\bar{\mu}\rho\bar{S}C_D} + \left[\frac{V^2-2e}{V^2+2e}\right]\frac{2\cos\gamma}{\bar{\beta}\rho\bar{S}V^3C_D}\right\} + \frac{\bar{c}_2}{V^2} \qquad (40)$$

$$p'_\gamma = \frac{[V^2+2e]^2\cos\gamma}{2\bar{\mu}\,\rho\,\bar{S}\,V^3\,C_D}p_V + \frac{[V^4+4e^2]\sin\gamma}{2\bar{\mu}\,\rho\,\bar{S}\,V^4\,C_D}p_\gamma \qquad (41)$$

Note that the atmospheric density, ρ, is a function of V and e; so that the rate of change of ρ with V directly enters the co-state equation for p_V.

$$\frac{\partial}{\partial V}\left\{\frac{1}{\rho(V,e)}\right\} = -\frac{4\bar{\mu}V}{\bar{\beta}(V^2+2e)^2}\frac{1}{\rho(V,e)} \qquad (42)$$

From Pontryagin's minimum principle, the minimal cost occurs when the Hamiltonian is minimized within the allowable control set while the state and co-state are on the optimal trajectory; i.e., minimize $\mathcal{H}(V^*,\gamma^*,C_L,p_V^*,p_\gamma^*,e)$ over the allowable set of C_L's. First consider the optimal C_L with no constraints on C_L.

$$\frac{\partial \mathcal{H}}{\partial C_L}(V^*,\gamma^*,C_L,p_V^*,p_\gamma^*,e) = -2K\frac{C_L}{C_D^2}h_1^* - 2K\left(\frac{C_L}{C_D}\right)^2 h_2^* + \frac{h_2^*}{C_D} \qquad (43)$$

where h_1^* and h_2^* are (37) and (38) respectively, evaluated on the optimal trajectory.

Setting (43) to zero results in

$$C_L = -\frac{h_1^*}{h_2^*} \pm \sqrt{\left(\frac{h_1^*}{h_2^*}\right)^2 + \frac{C_{Do}}{K}} \qquad (44)$$

Note that the negative sign in front of the discriminant always produces a negative C_L, while the positive sign results in a positive C_L value. Observation of (36) shows that the minimal $\mathcal{H}$ occurs when

$$h_2^* C_L \leq 0 \quad , \qquad (45)$$

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since C_D is always positive. It follows that the root which corresponds to the minimum is given by

$$C_L^* = -\left(\frac{h_1^*}{h_2^*}\right) - \mathrm{sgn}\,(h_2^*)\sqrt{\left(\frac{h_1^*}{h_2^*}\right)^2 + \frac{C_{Do}}{K}} \quad . \tag{46}$$

Now, suppose that C_L is limited to be within $[C_{Lmin}, C_{Lmax}]$, where C_{Lmax} is positive and C_{Lmin} is negative. The minimal C_L for this case is easily obtained by analyzing the gradient $\partial \mathcal{H} / \partial C_L$ in (43). Rewriting this gradient in the form

$$\frac{\partial \mathcal{H}}{\partial C_L} = -\frac{(Kh_2^*)C_L^2 + (2Kh_1^*)C_L - h_2^* C_{Do}}{C_D^2} \quad . \tag{47}$$

Suppose h_2^* is negative, then the gradient has two zeroes, C_{L-} and C_{L+} corresponding to the sign selected in (44). Note that C_{L-} is negative while C_{L+} is positive. The basic shapes of the gradient and the Hamiltonian lead to a value of $C_L^* \geq 0$ which is limited by C_{Lmax}. A similar analysis for the case of h_2^* being positive results in

$$C_L^* = \begin{cases} C_L, & C_{Lmin} \leq C_L \leq C_{Lmax} \\ C_{Lmax}, & C_L > C_{Lmax} \\ C_{Lmin}, & C_L < C_{Lmin} \end{cases} \tag{48}$$

where C_L is given by (46).

The necessary conditions for the optimal trajectory are given by the state equations (20), (21), the co-state equations (40), (41), and the control equations (46), (48), with the initial and final state satisfying (27) and (28). Thus, the necessary conditions specify the two-point-boundary-value problem given above. The sufficiency of these conditions is not treated here.

5. NUMERICAL RESULTS

To obtain the optimal guidance trajectories resulting from the optimal control problem posed, the necessary conditions were solved using a standard two-point-boundary-value problem (TPBVP) solver. The TPBVP solutions were obtained with shooting techniques [16] using the OPTSOL software package.

The problem considered was a typical coplanar orbit transfer from high earth orbit to low earth orbit using aeroassist to achieve the maneuver. The initial circular orbit is at an altitude of 22,366 km over the earth surface, while the low earth orbit is at an altitude of 715.6 km. The atmospheric entry conditions resulting from this initial orbit were selected to be a speed of 10 km/sec and flight path angle of -6° by choosing a target perigee at a radial distance of 6406.5 km from the earth center.

The atmospheric exit conditions are specified by the speed of 8 km/sec and flight path angle of 0.01 rad or 0.57° to ensure a slightly positive flight path angle to exit the atmosphere. The top of the atmosphere was selected at 127 km or a radial distance of 6498 km. The atmospheric scale height was set at 7.5 km with the zero altitude density ρ_o corresponding to 7.769×10^{10} kg/km^3.

The vehicle parameters C_{Do} and K were set respective values of 0.05 and 1.4. The maximum lift-to-drag ratio for the vehicle was 1.9. The effective mass to vehicle area ratio used was 300 kg/m^2.

Since the initial condition of the state, y, is specified, solving the TPBVP consists of finding the initial co-state values which will drive the 4th order state/co-state system of differential equations to the desired final state. Since the co-state equations integrated forward, are usually unstable, the solution of TPBVP's is a difficult problem. Significant convergence problems were, in fact, encountered in solving the necessary conditions.

The sensitivity of the optimal initial co-state vector to changes in the problem parameters was found to be high. A consequence of this sensitivity is that the radius of convergence of the shooting algorithm for the problem considered was relatively low. So that when parameters such as the drag coefficient C_{D_o} or the scale height, $\bar{\beta}$, of the atmospheric density are varied by small amounts, the algorithm does not converge; this was found to be the case particularly at lower values of the scale height. Although a complete study was not made, multiple shooting did not significantly modify this situation. On the other hand, in many cases, the rate of convergence of the single shooting algorithm was fast, requiring under ten iterations.

A parametric study of the optimal guidance trajectories for different linear combinations of the heating rate versus the drag terms in the cost function was performed. The optimal trajectories obtained are shown in Figures 2-5. The proportions of the heating rate (squared) and drag were varied by fixing $\bar{c}_1$ at .001 while $\bar{c}_2$ takes on the values of 2.0, 1.0, 0.6 and 0.0. Recall that when $\bar{c}_2$ vanishes the cost function minimizes only the heating rate term. As $\bar{c}_2$ increases, the cost function contains greater proportions of the drag force term so that the shear stress on the skin is also included as as objective.

The basic features of all the trajectories are similar. The speed is reduced from 10 km/sec to 8 km/sec slowly at first, then at a higher rate until reaching approximately 8.2 km/sec. At that point, the curve flattens considerably, slowly moving towards its final value at 8. Similarly, the flight path angle is increased until it reaches nearly $+ 1.4°$. At this point, a rather sharp or decisive reversal of the trend brings the flight path angle to a flat curve until atmospheric exit.

The heating rate increases as the vehicle dips into the atmosphere. However, it remains under 100 W/cm^2 throughout the atmospheric maneuver. This level is satisfactory, as much higher rates can be accommodated. The coefficient of lift starts near a value of two and remains there initially until it drops and settles near a negative value of -0.5. The trajectories shown here correspond to the unconstrained control case. Due to convergence

problems and time constraints, the constrained control cases were not obtained.

As the proportion of the heating rate versus drag is increased, the essential character of the trajectory remains unchanged. The main difference is seen in the duration of the maneuver which increases as the drag term is phased out. Also note that the final flat portion of the trajectory is lengthened while the prior portion is slightly shortened in time. However, the heating rate is largely unchanged and remains safely under 100 W/cm^2 in all the trajectories. The altitude profile tends to become slightly more flat at the end of the maneuver when $\bar{c}_2 = 0$.

6. SUMMARY

The atmospheric portion of the trajectories for the aeroassisted coplanar orbit transfer have been investigated. The equations of motion for the problem are expressed using a new reduced order model using total vehicle energy, kinetic plus potential, as the independent variable rather than time. The order reduction is achieved analytically without an approximation of the vehicle dynamics.

In this model, the problem of coplanar orbit transfer is seen as one in which a given amount of energy must be transferred from the vehicle to the atmosphere during the trajectory without overheating the vehicle. An optimal control problem is posed where a linear combination of the integrated square of the heating rate and the vehicle drag is the cost function to be minimized. The necessary conditions for optimality are obtained. These result in a 4th order two-point-boundary-value problem.

A parametric study of the optimal guidance trajectory in which the proportion of the heating rate term versus the drag varies is made. The problem considers transferring the vehicle from an orbit at an altitude of 22,366 km to one at an altitude of 715.6 km in a two impulse aeroassisted maneuver. Sensitivity and convergence problems of the shooting algorithm are discussed. Simulations of the guidance trajectories are presented.

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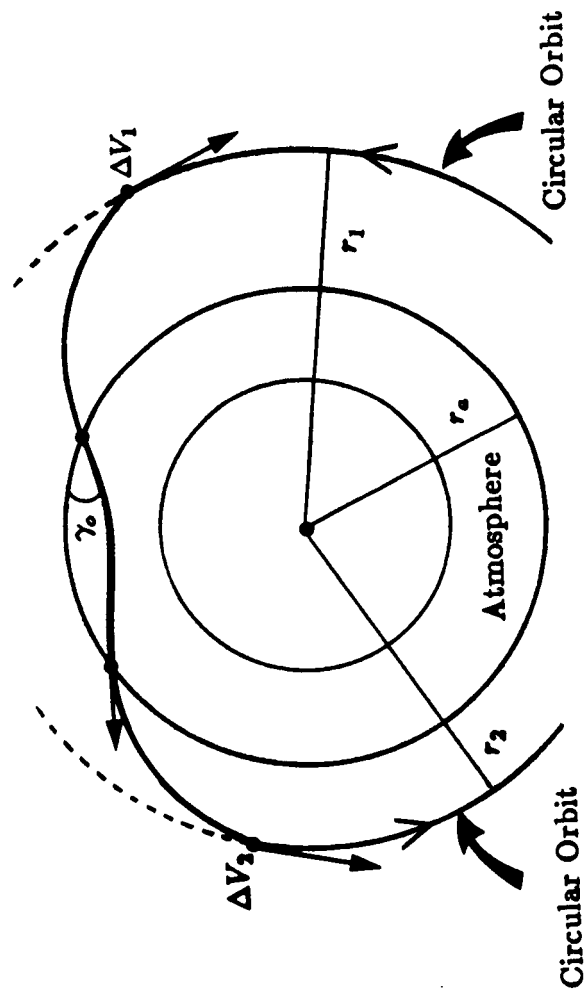
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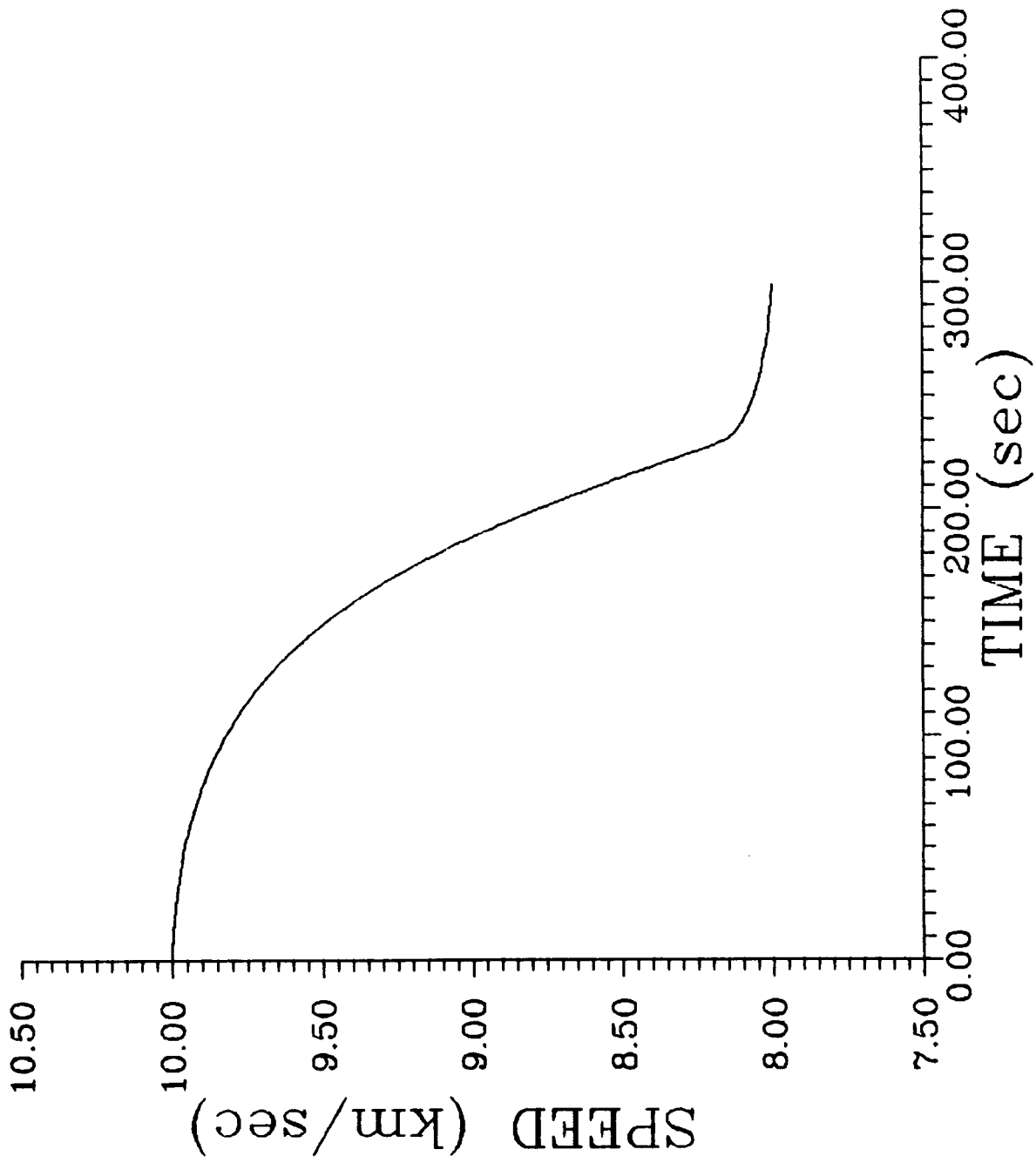
Figure 2.a. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$

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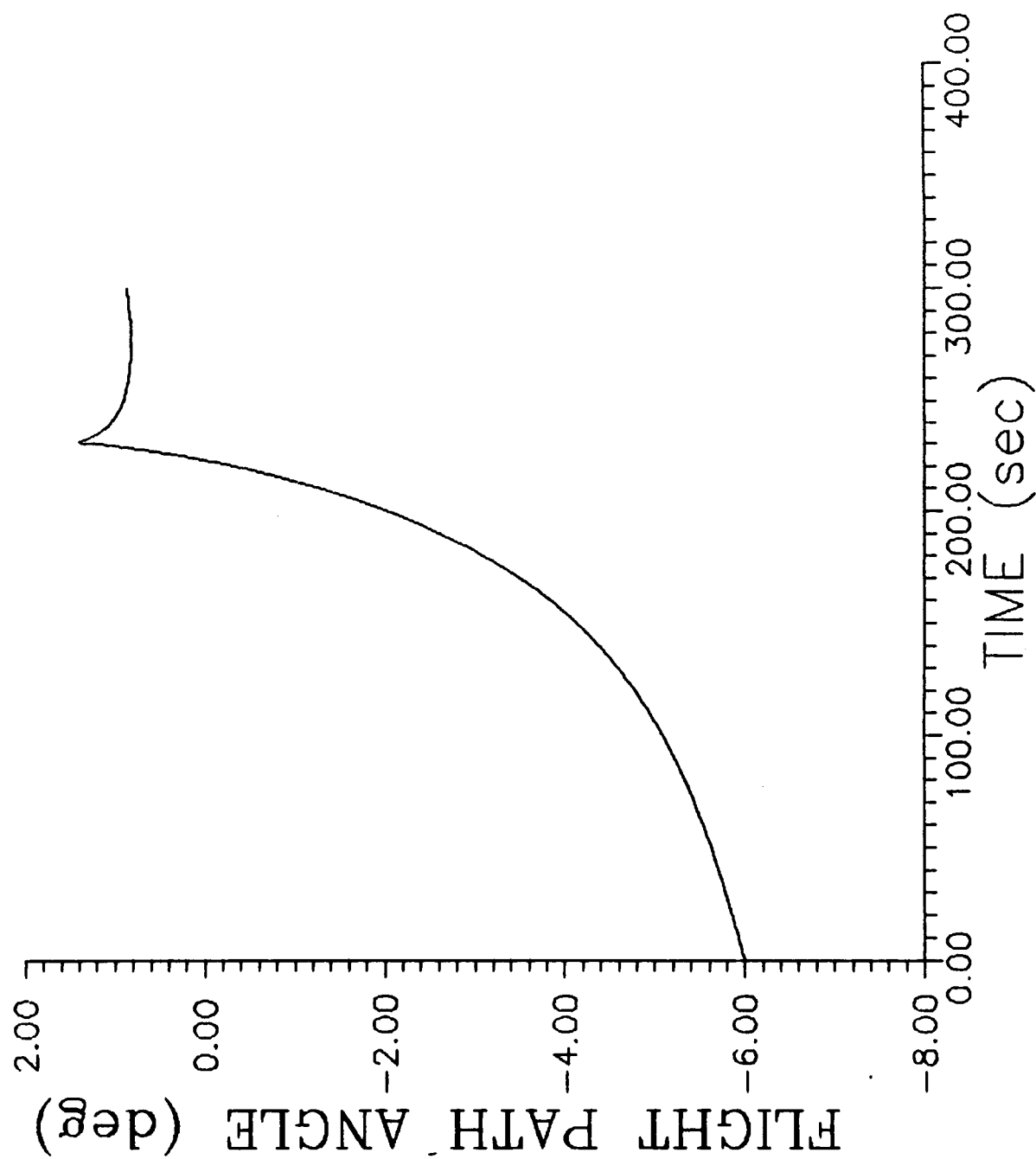
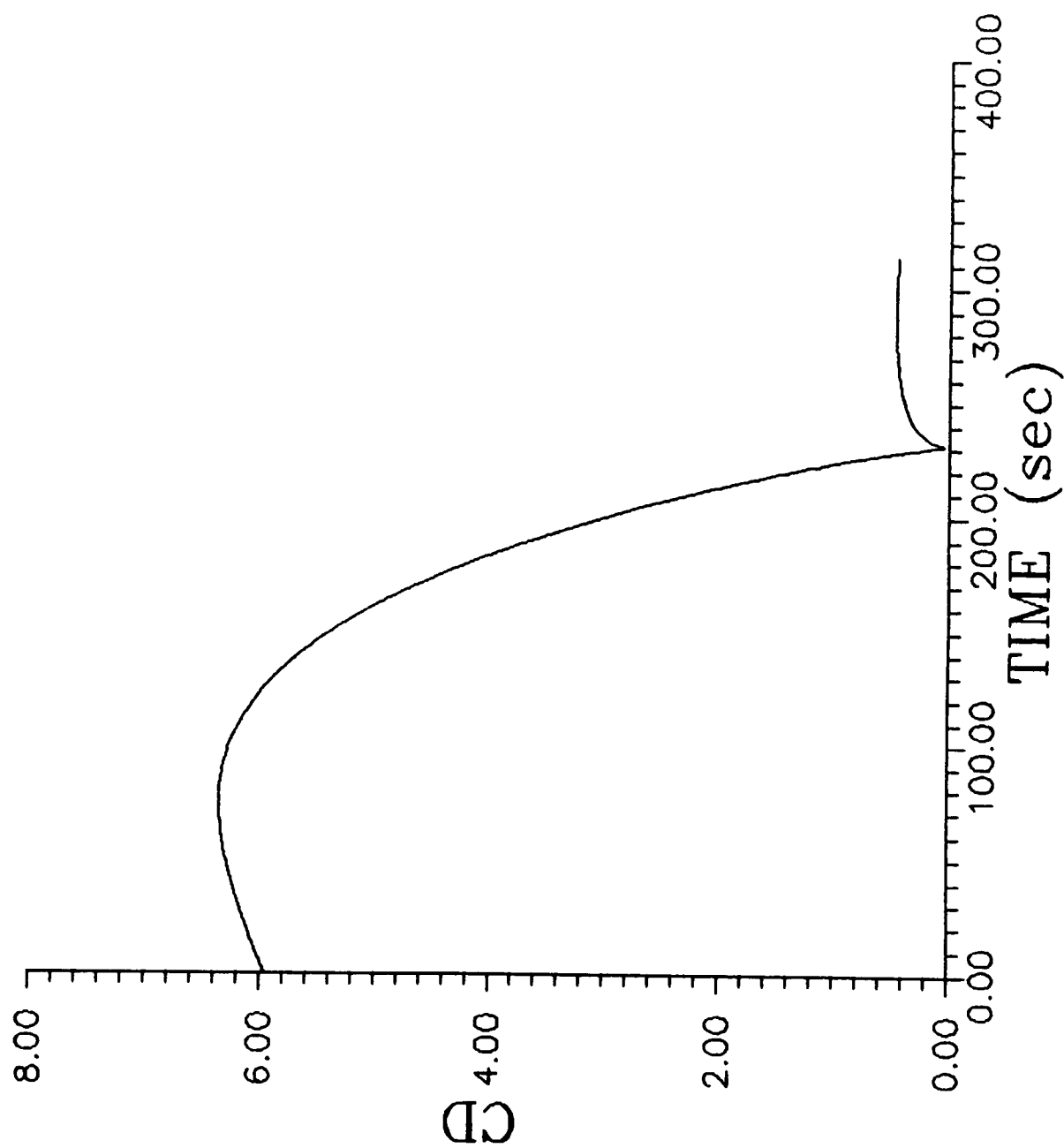Figure 2.b. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$

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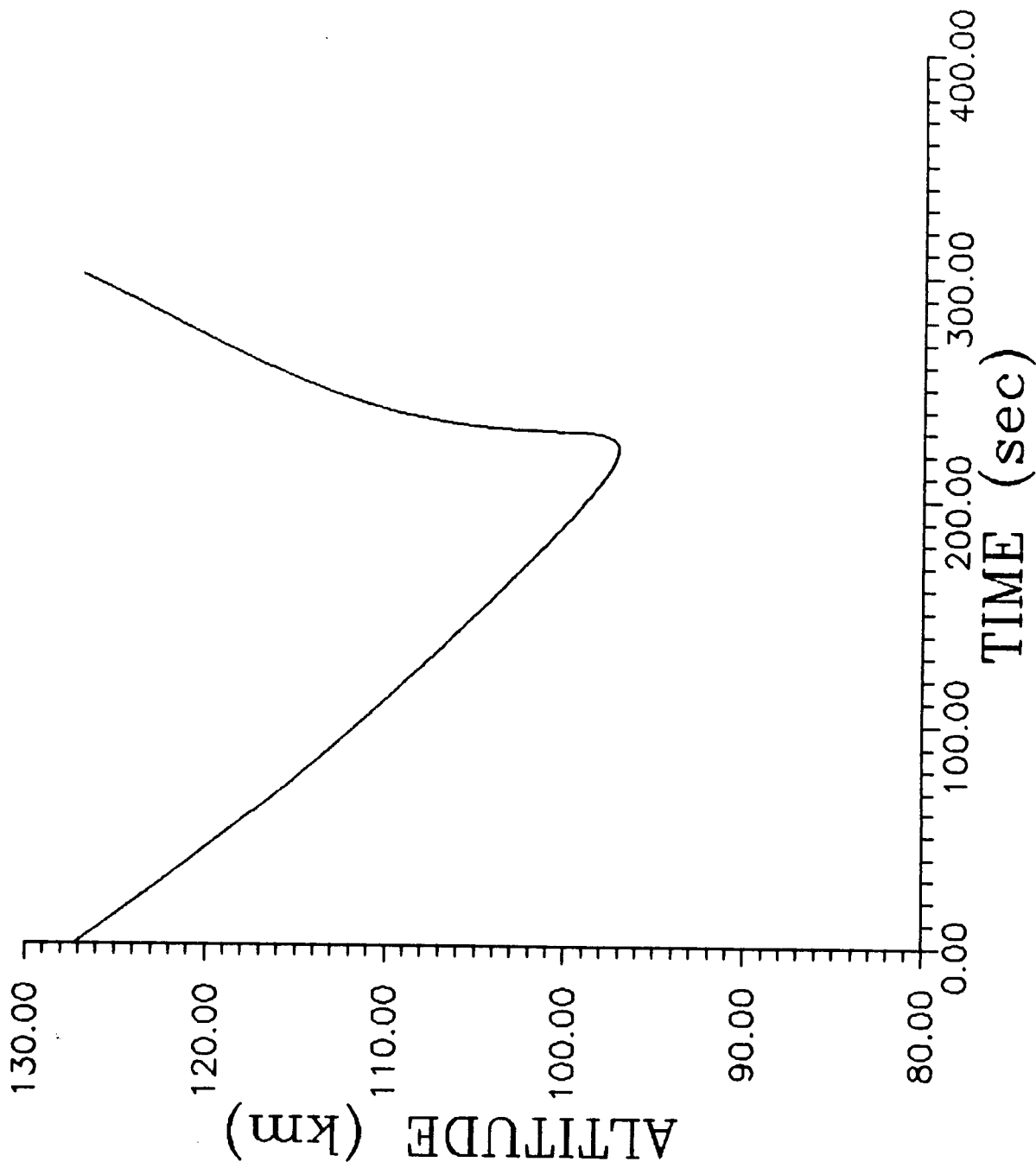
Figure 2.c. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$
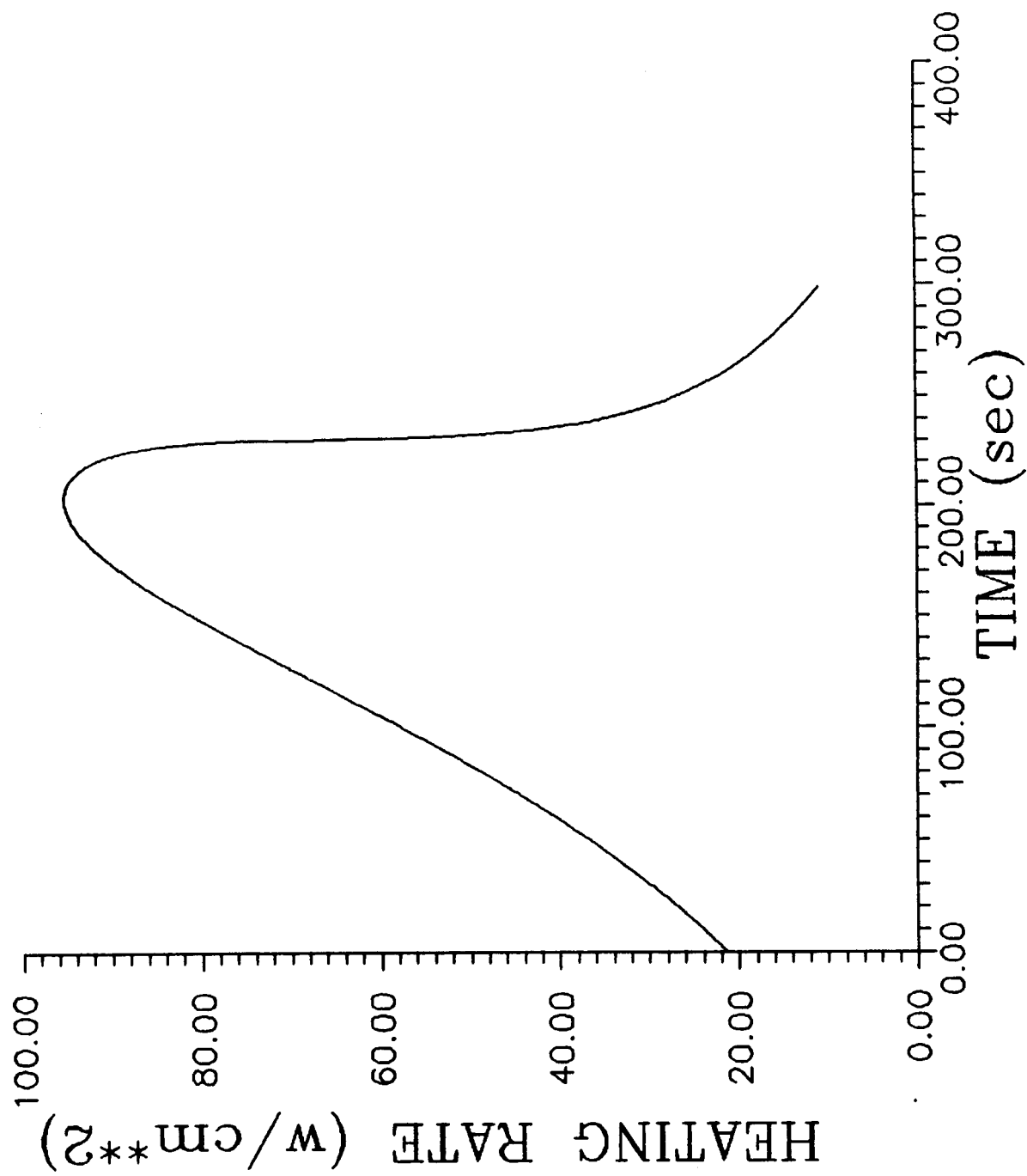
Figure 2.d. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$
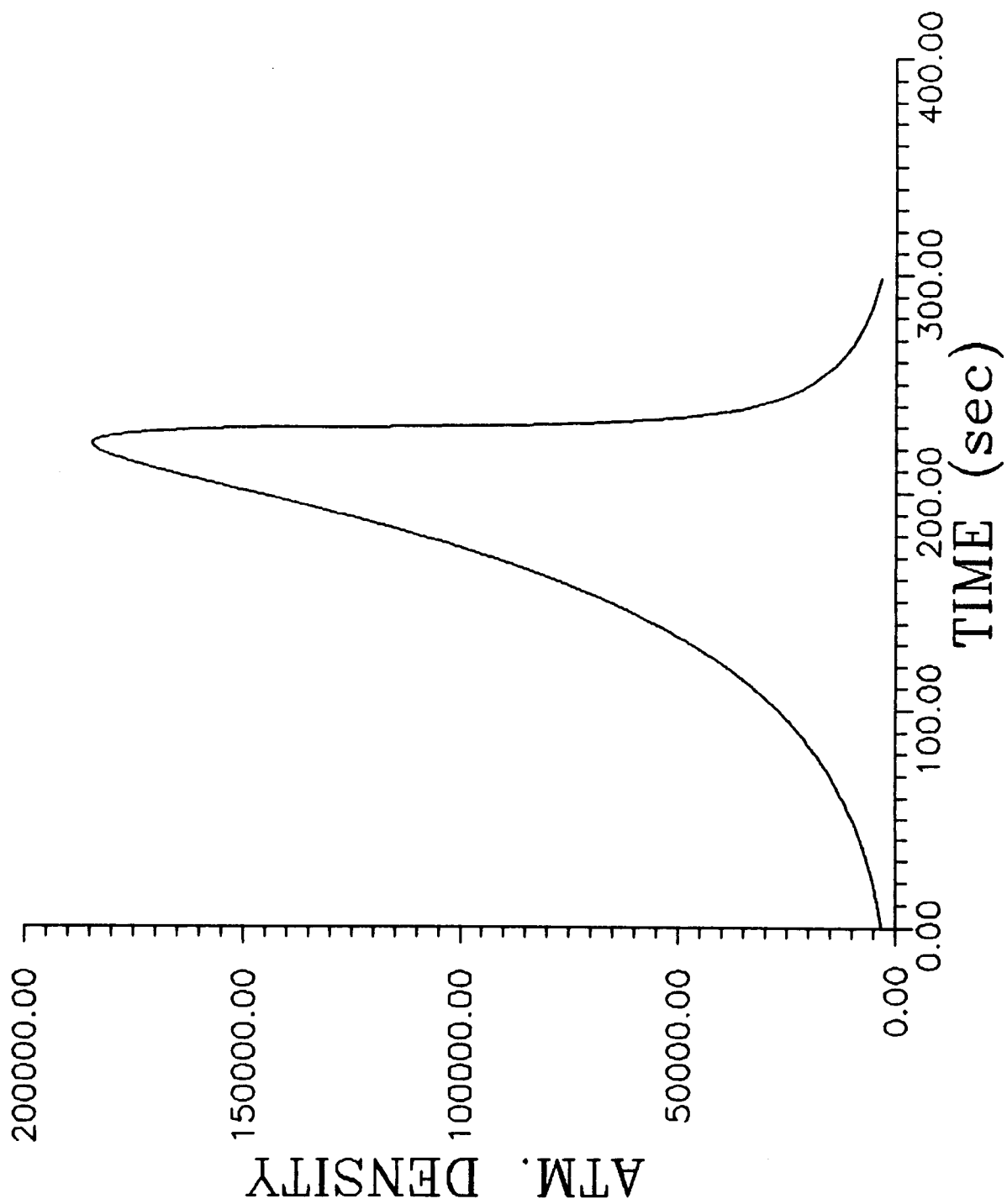
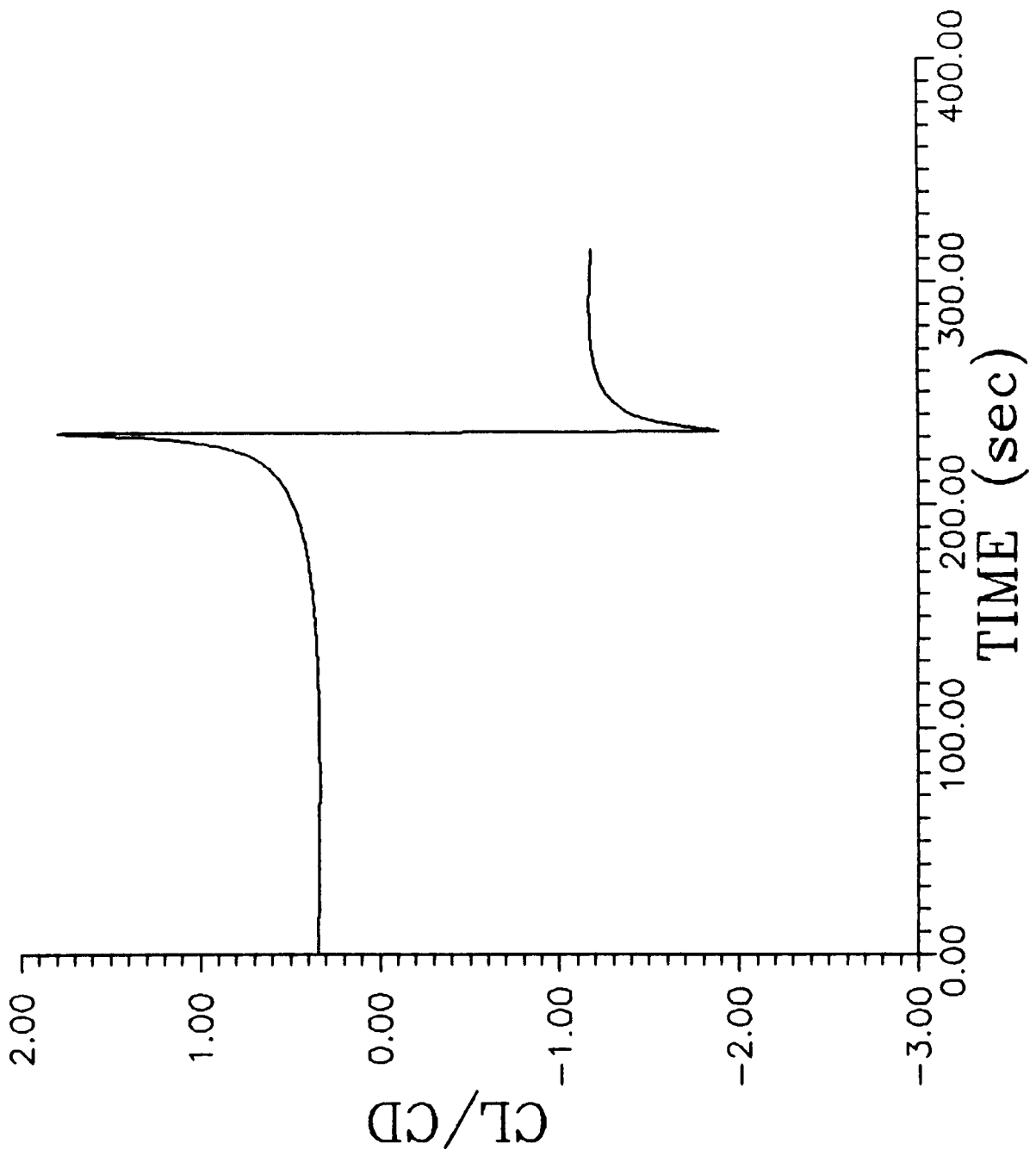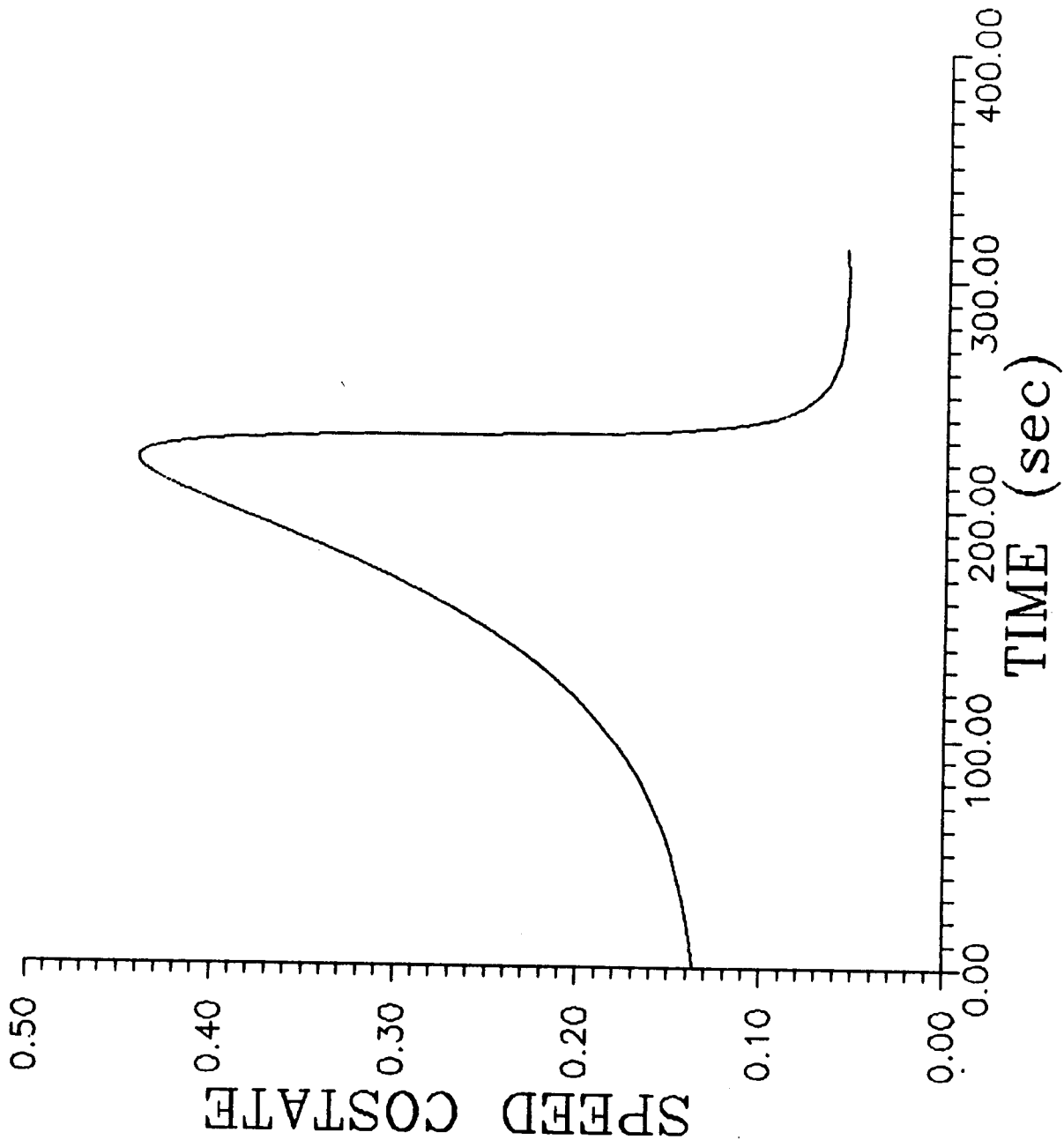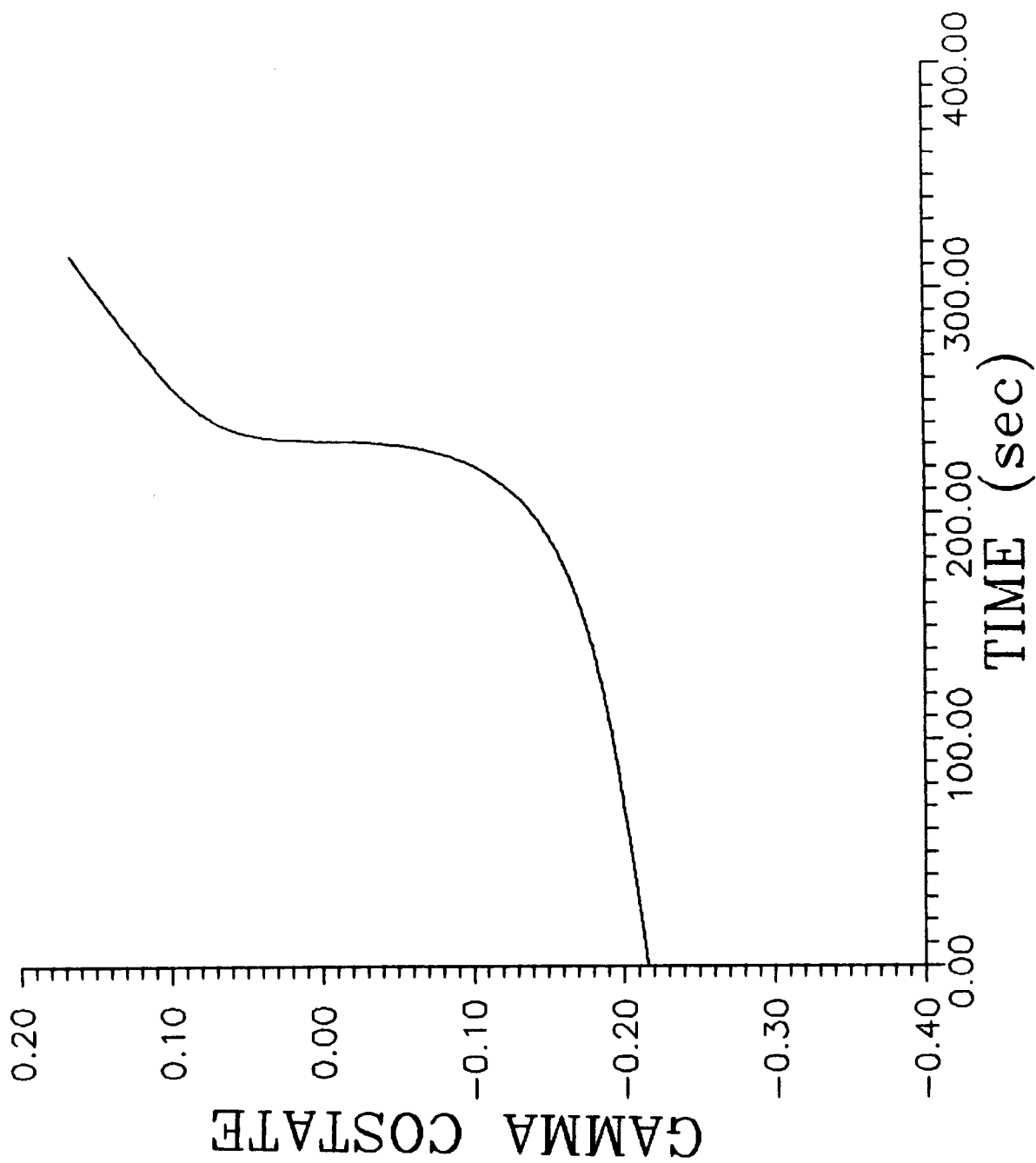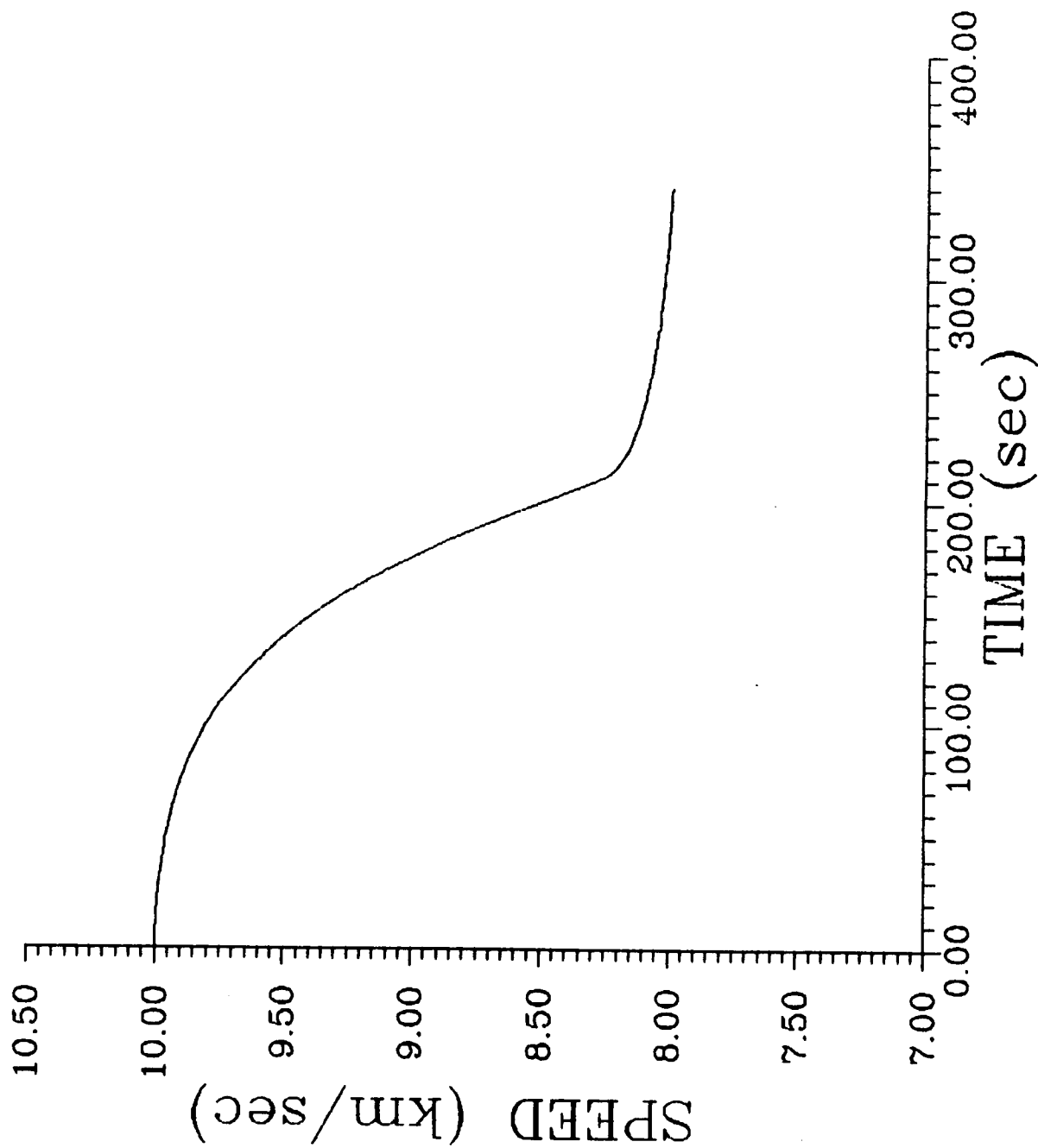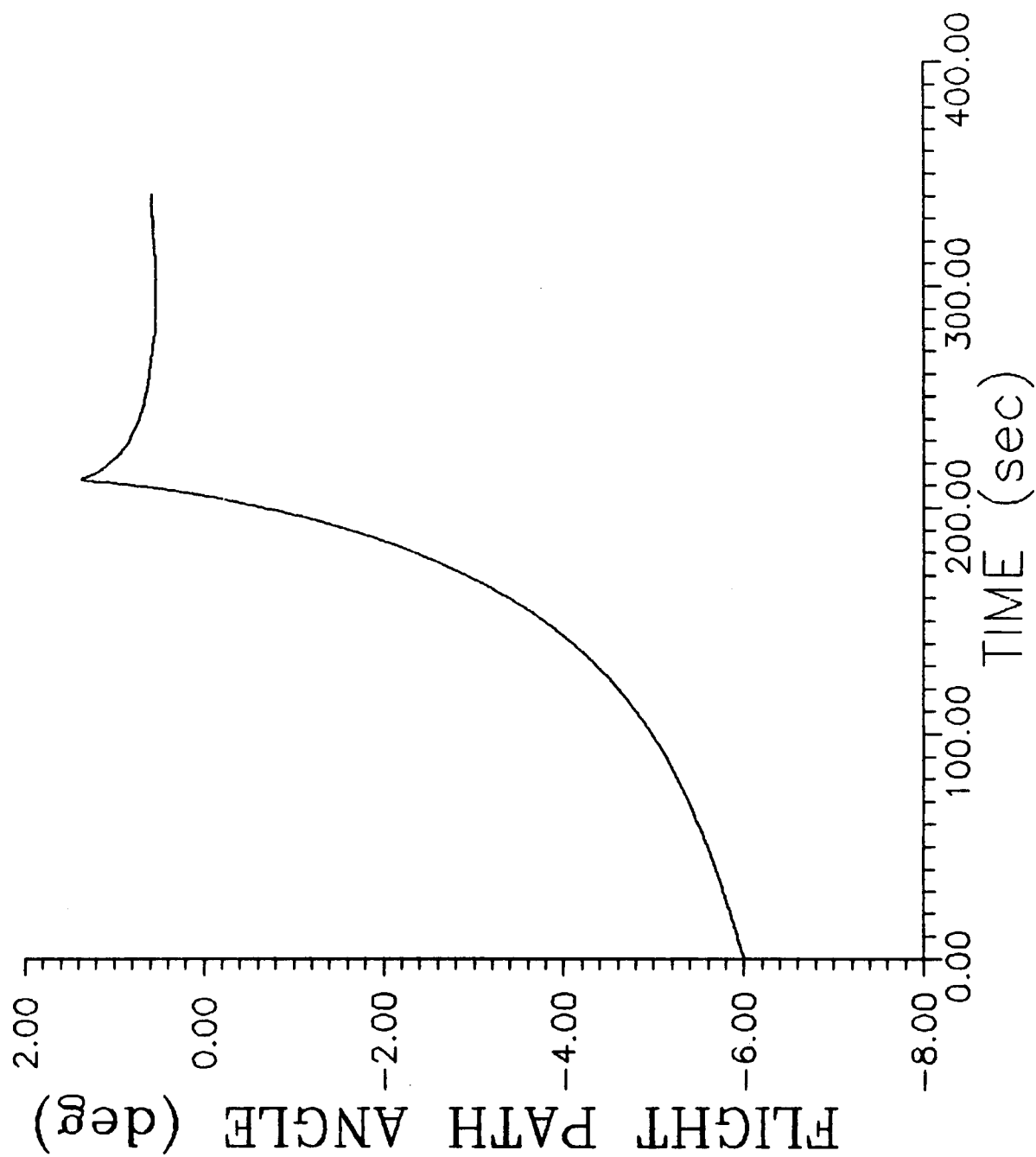Figure 2.e. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$
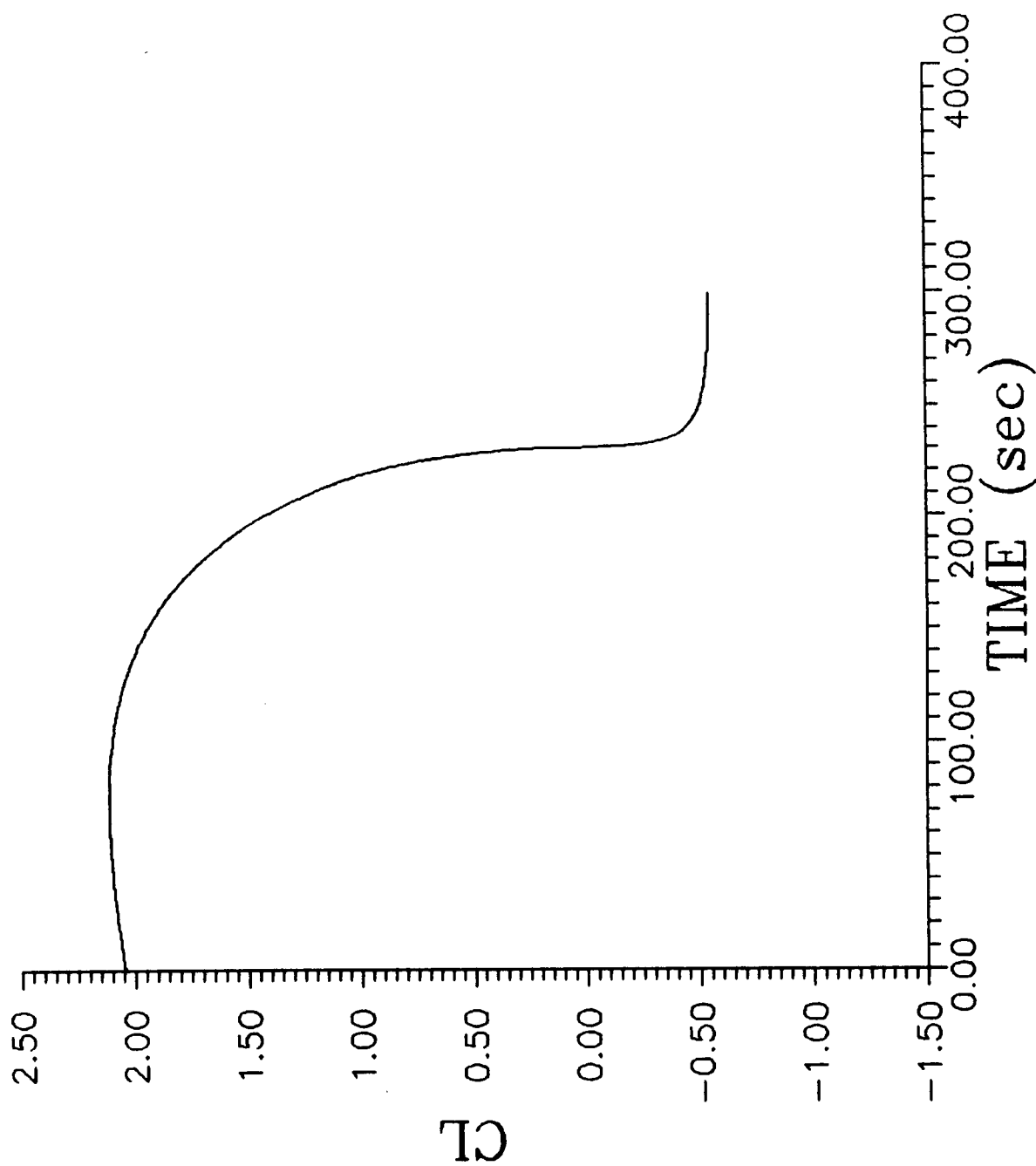
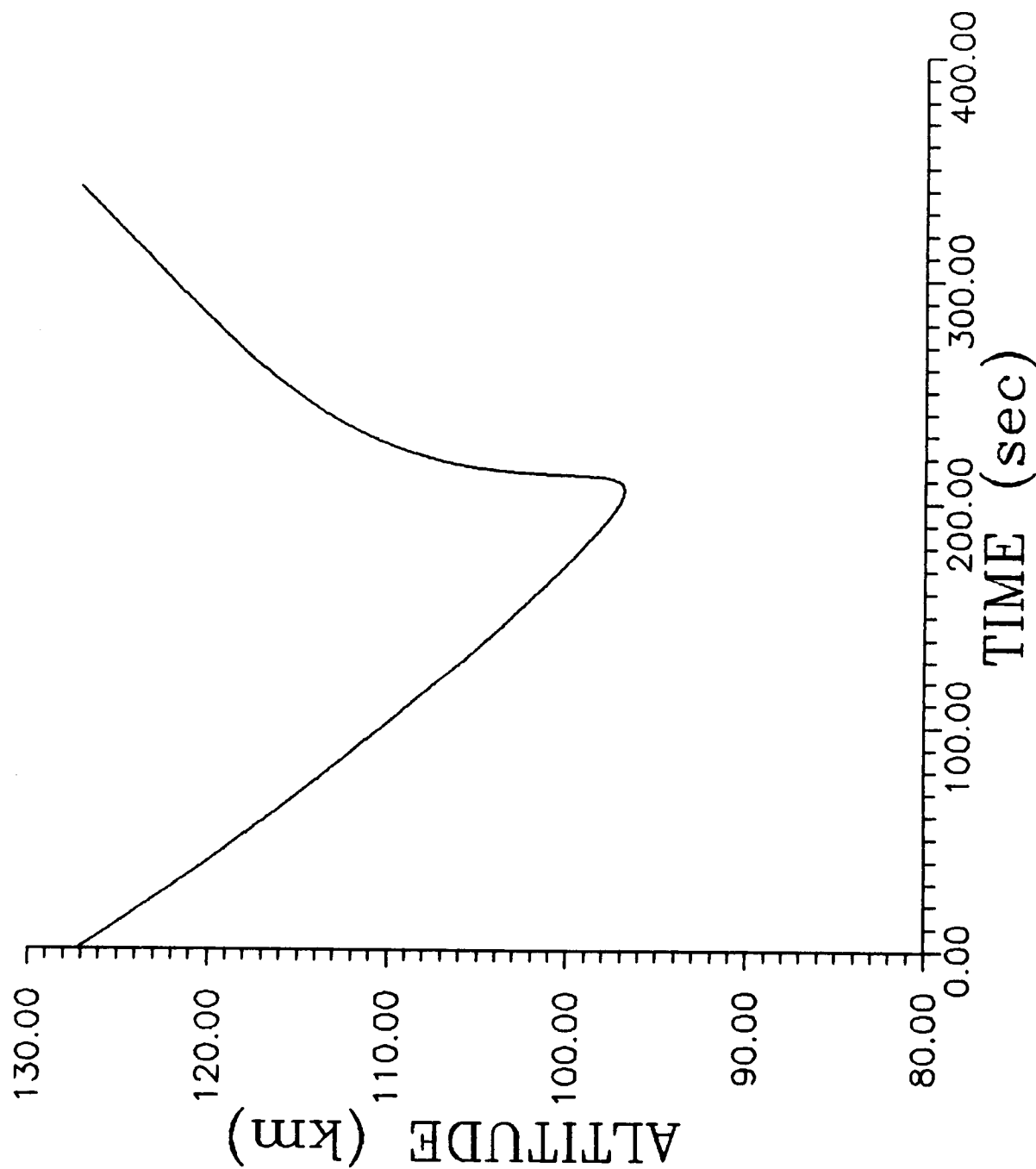Figure 2.f. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$

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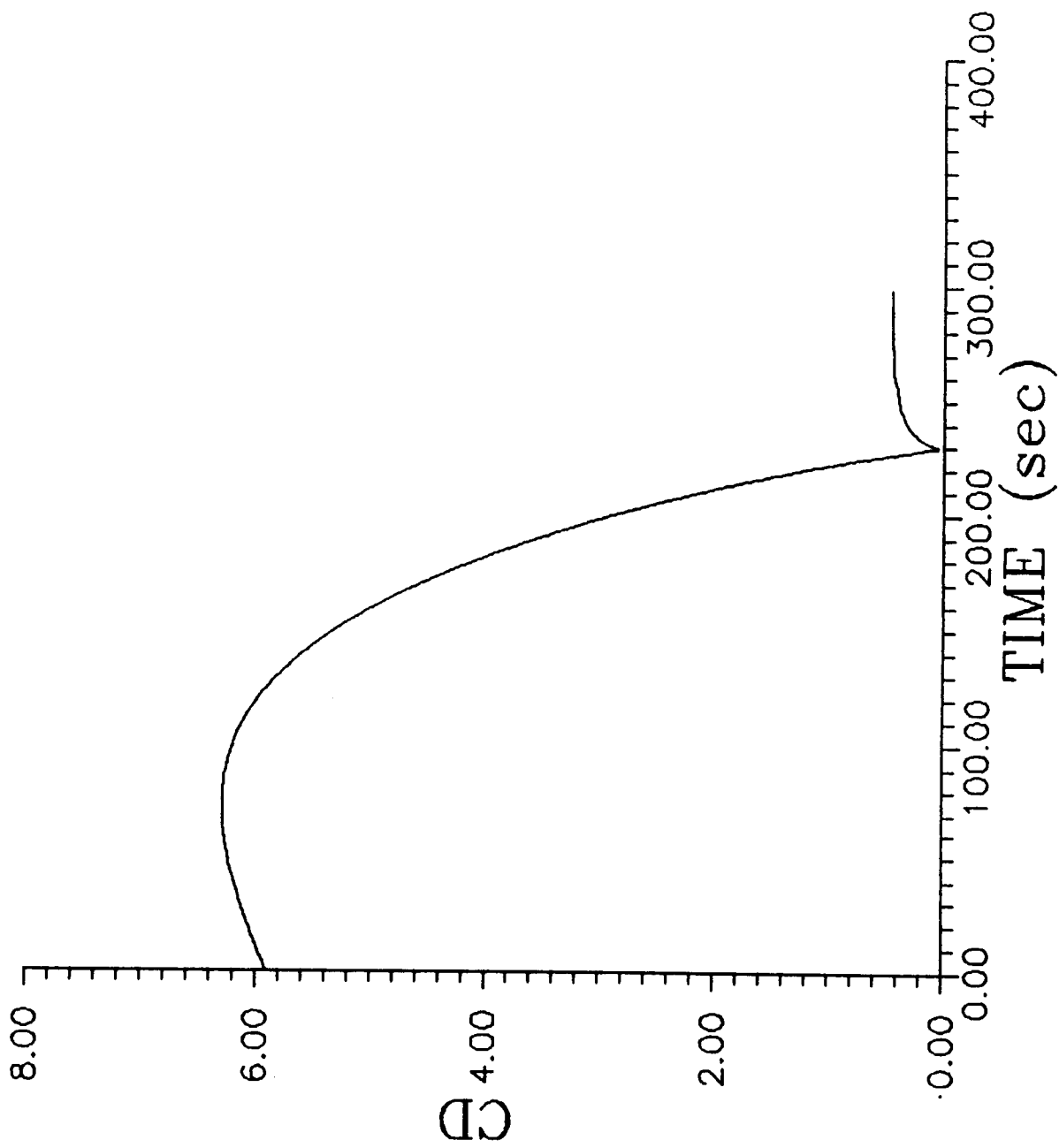
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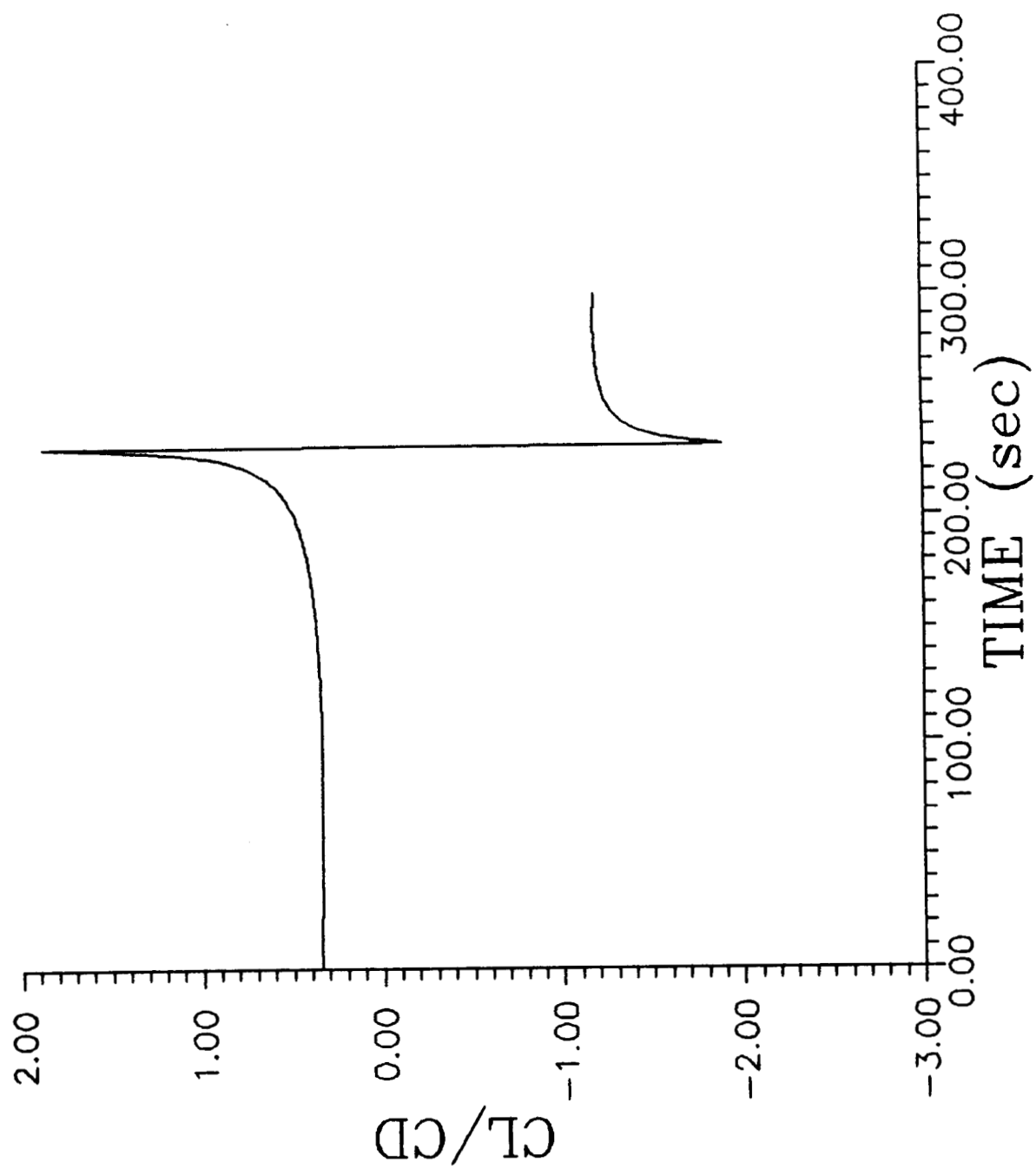
Figure 2.h. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 2.0$

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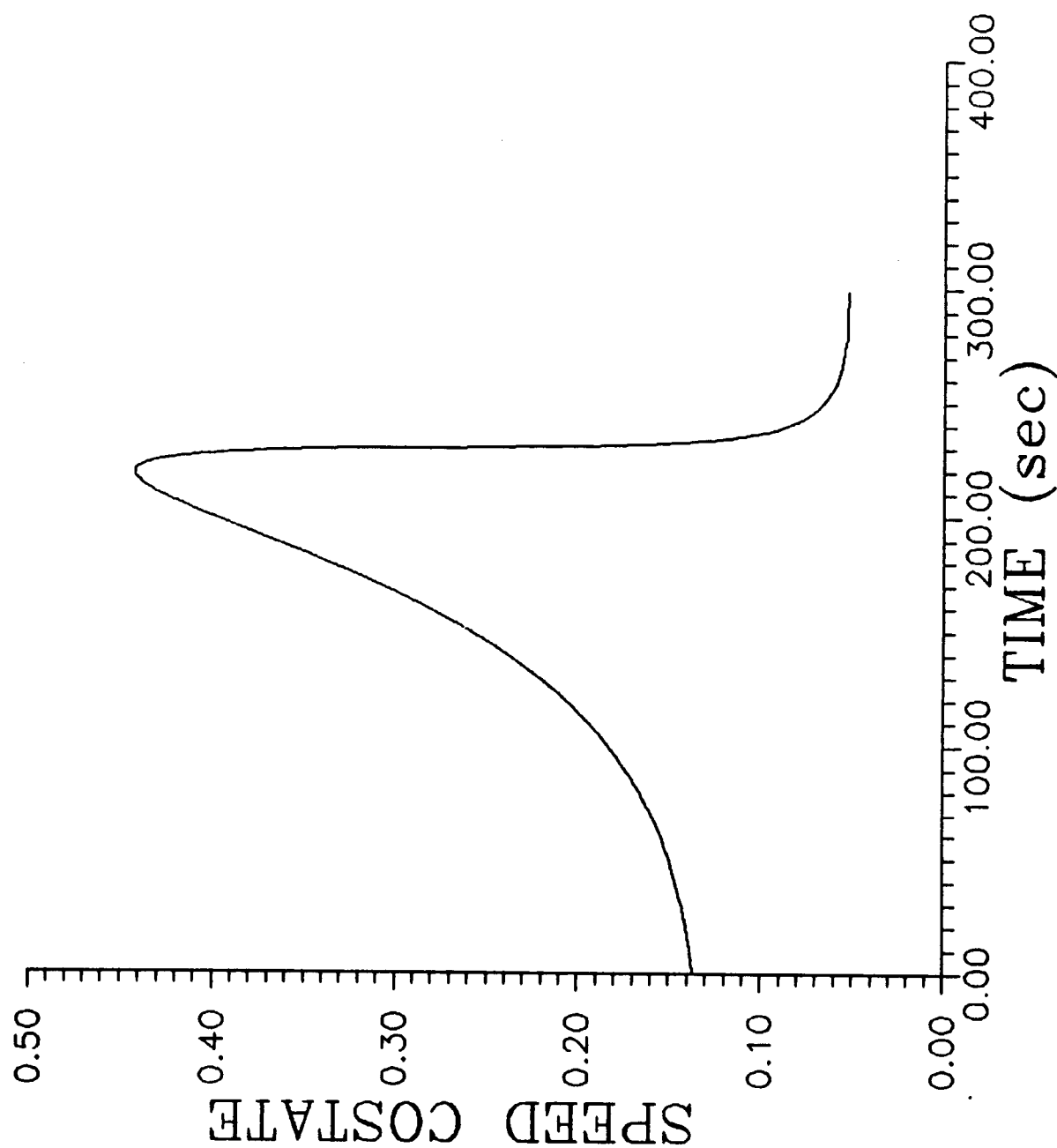
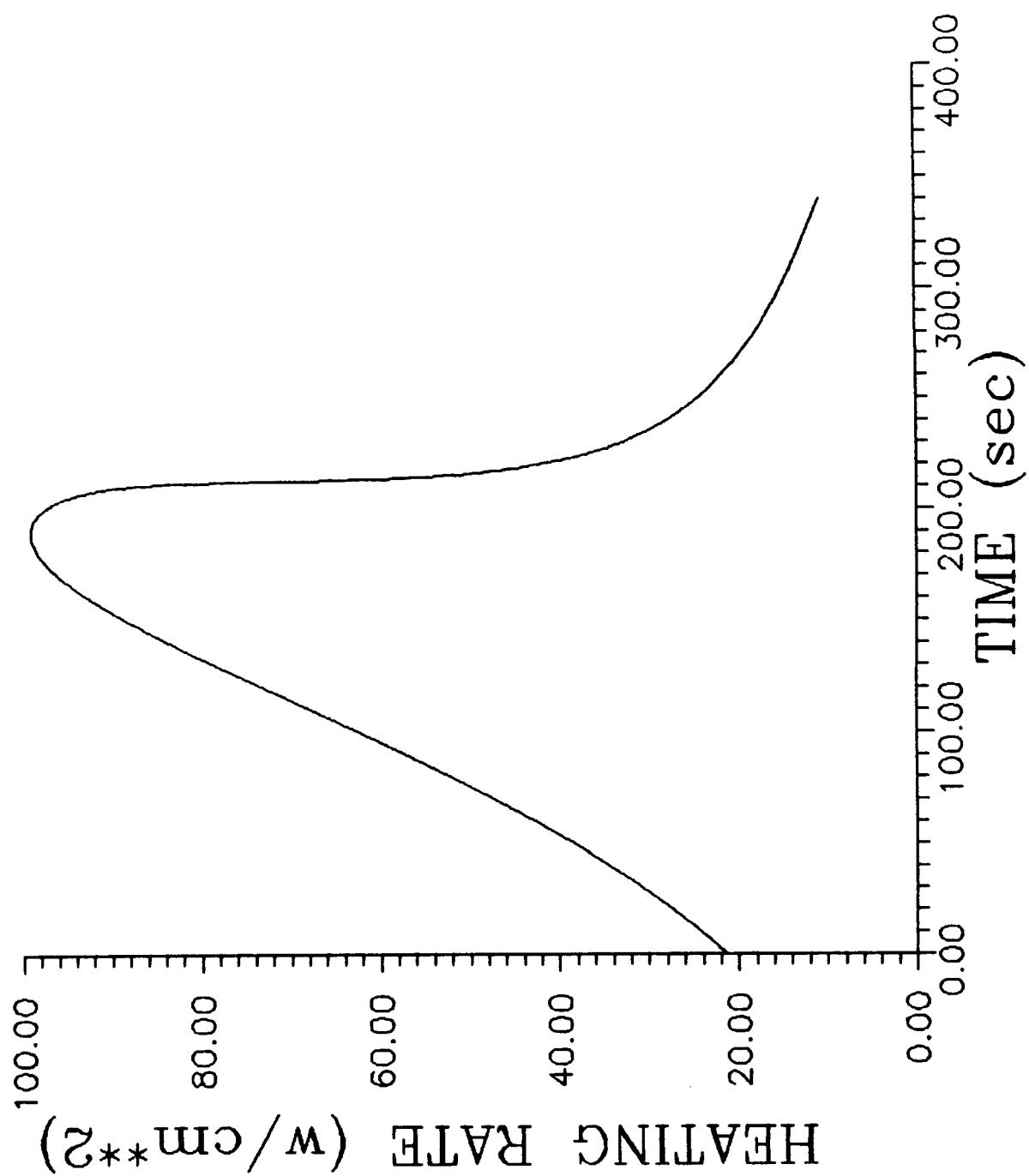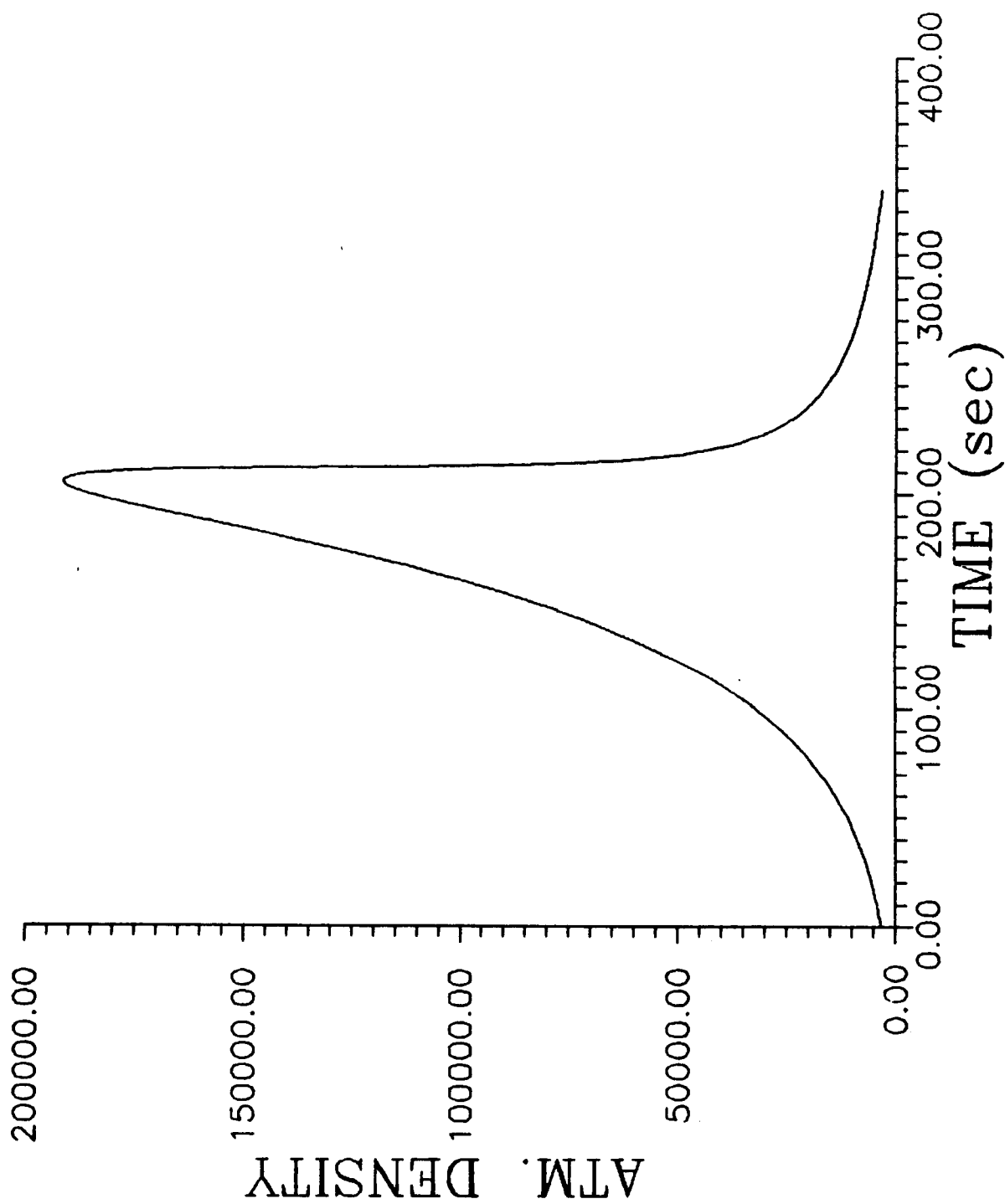Figure 2.i. Optimal Guidance Trajectory for $\bar{\varepsilon}_1 = .001, \bar{\varepsilon}_2 = 2.0$

31
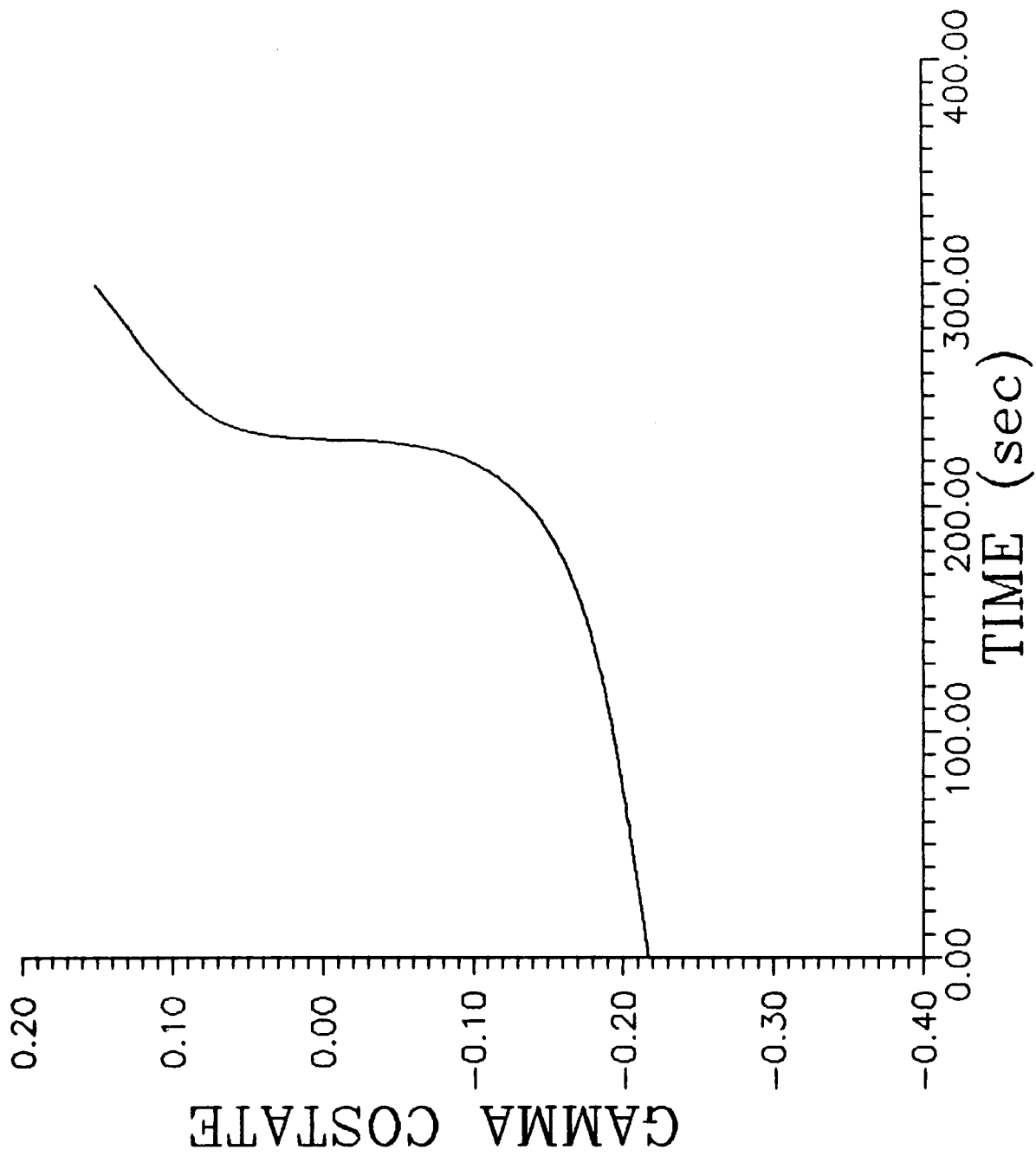
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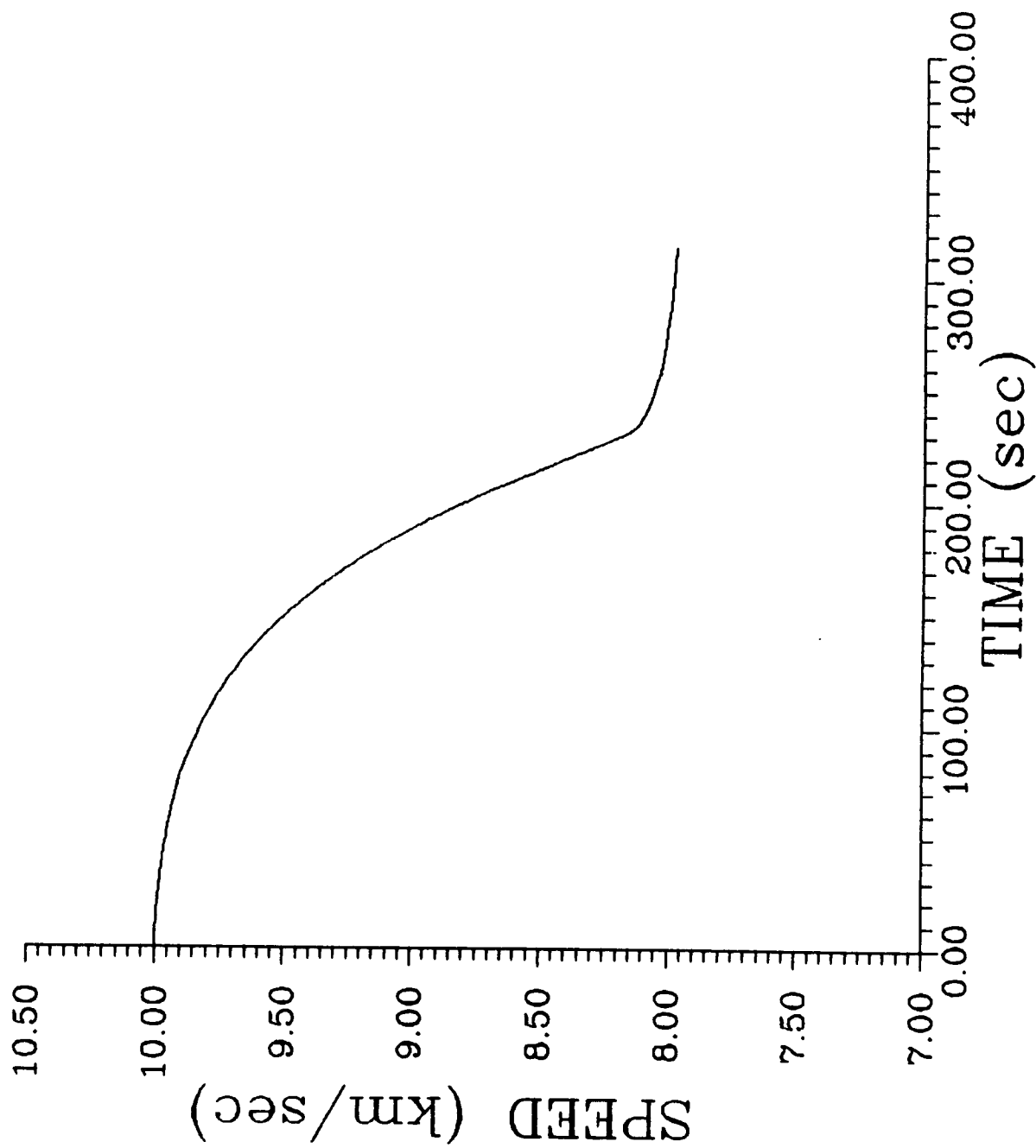
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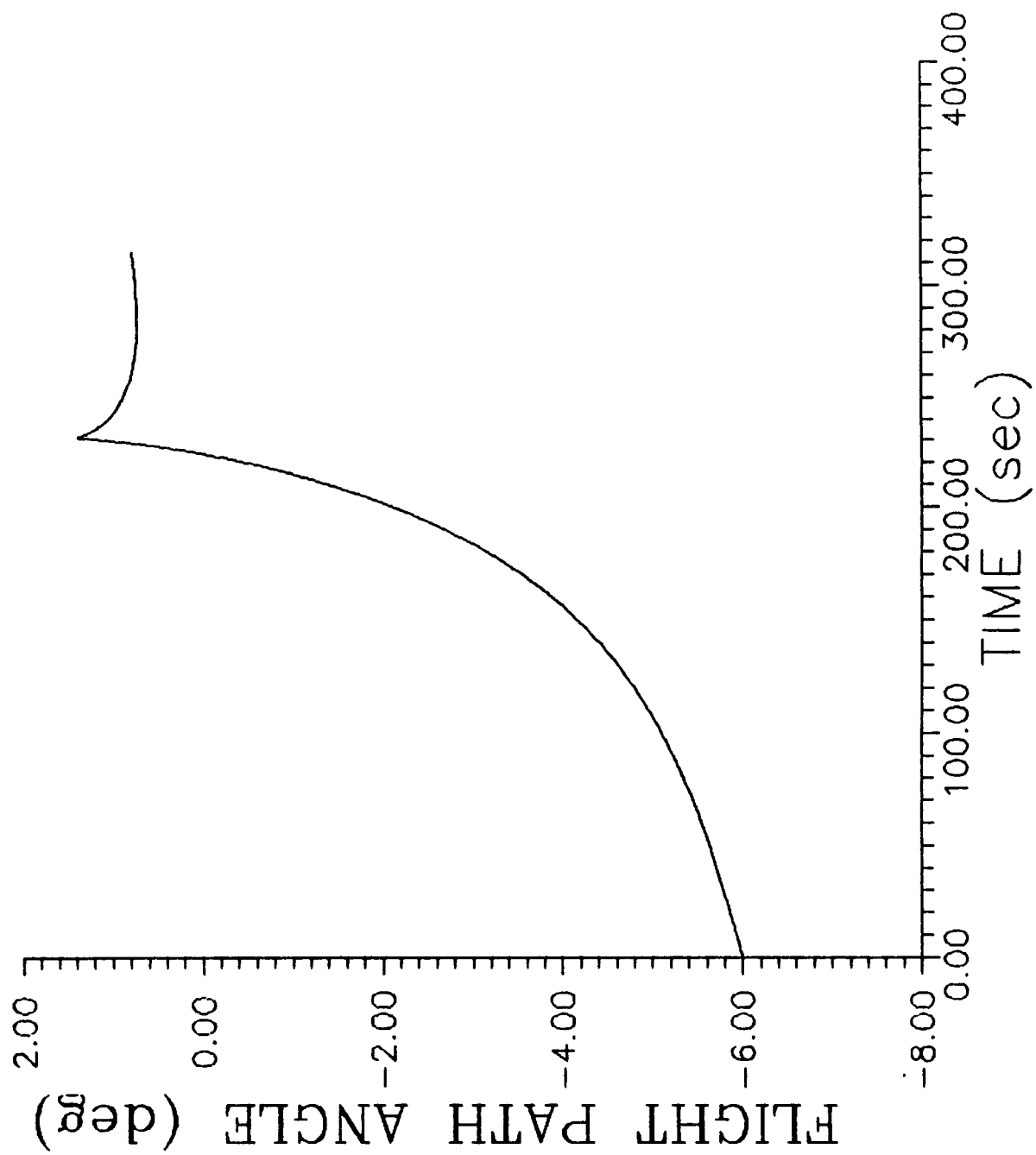
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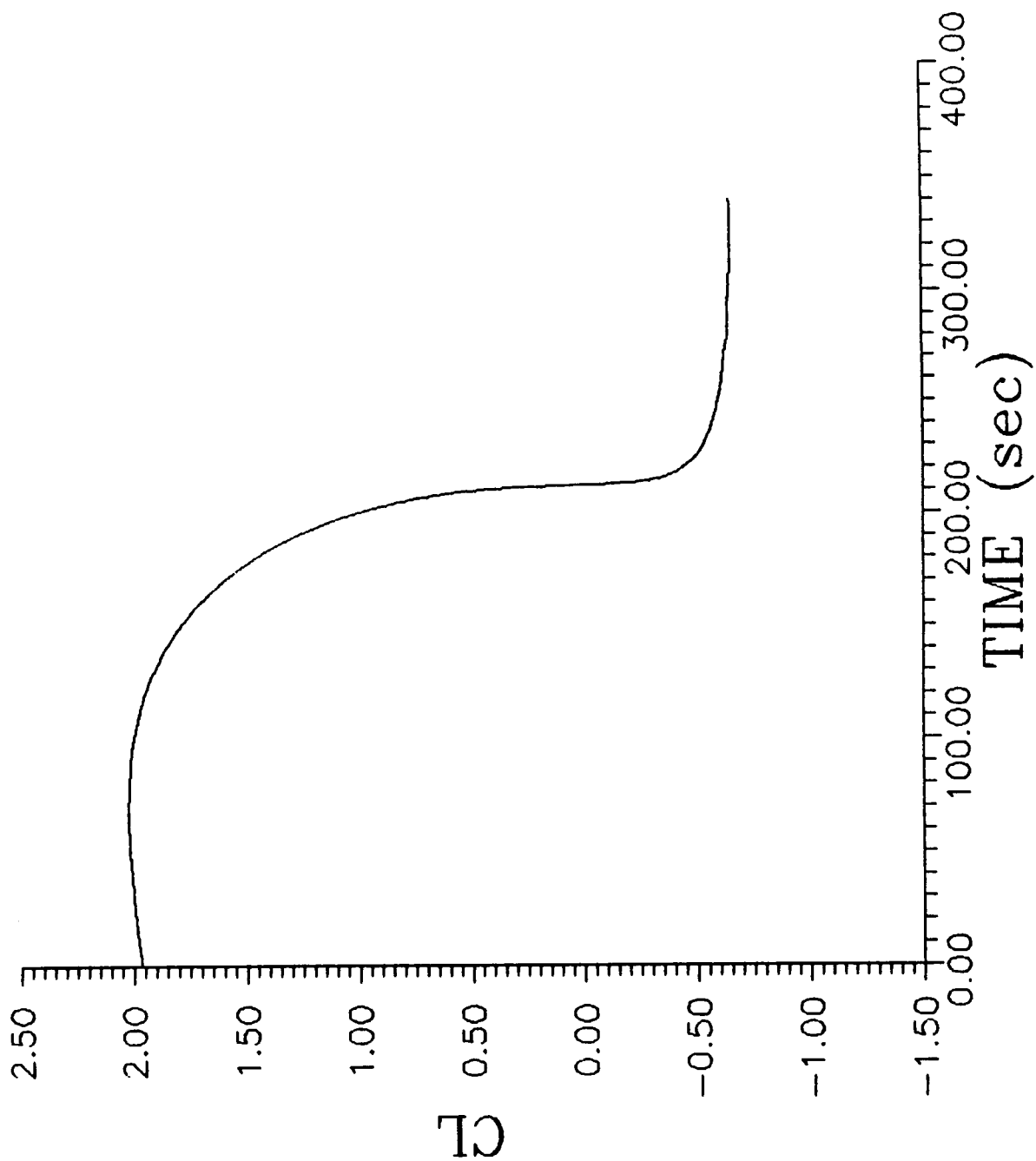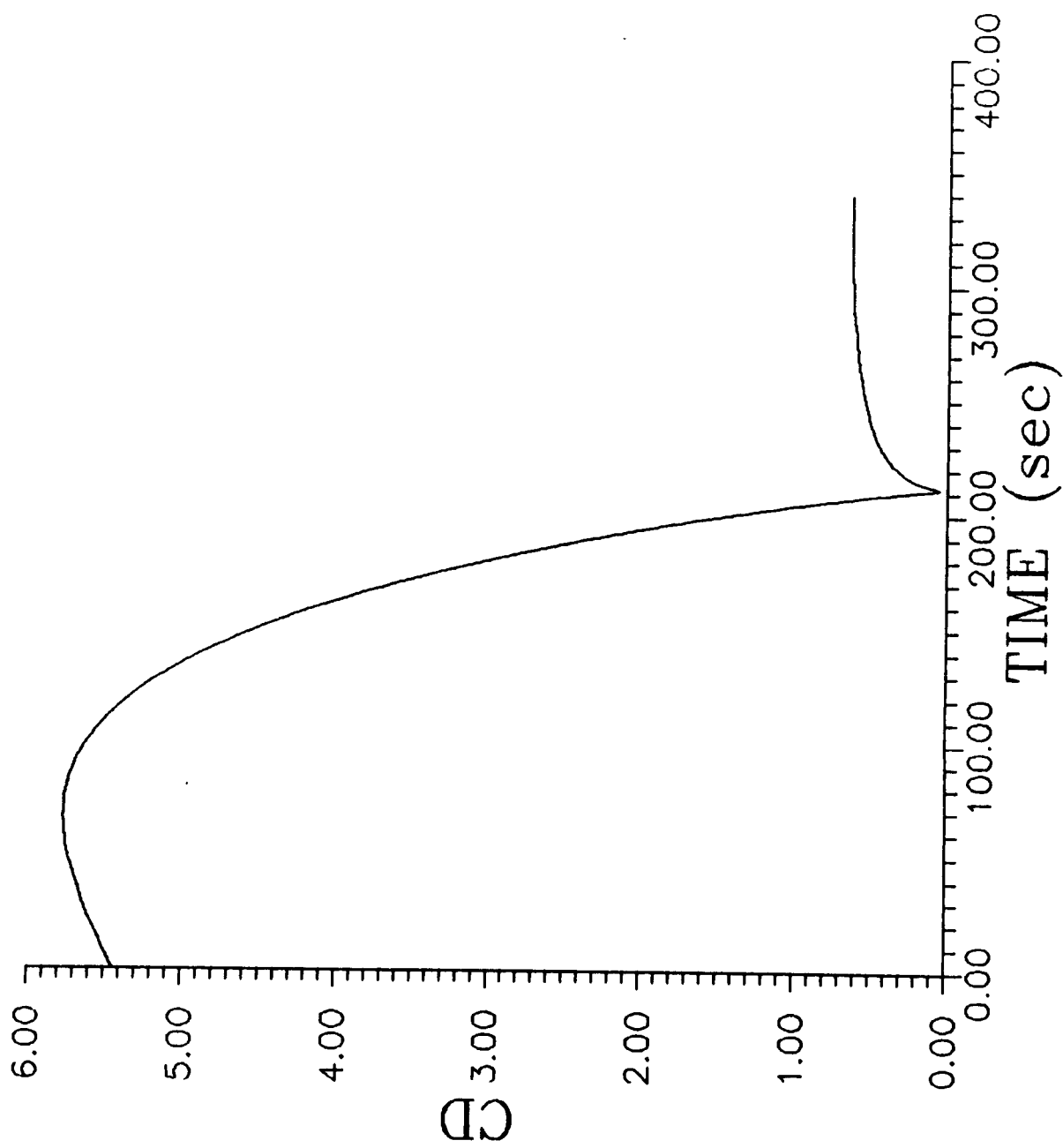
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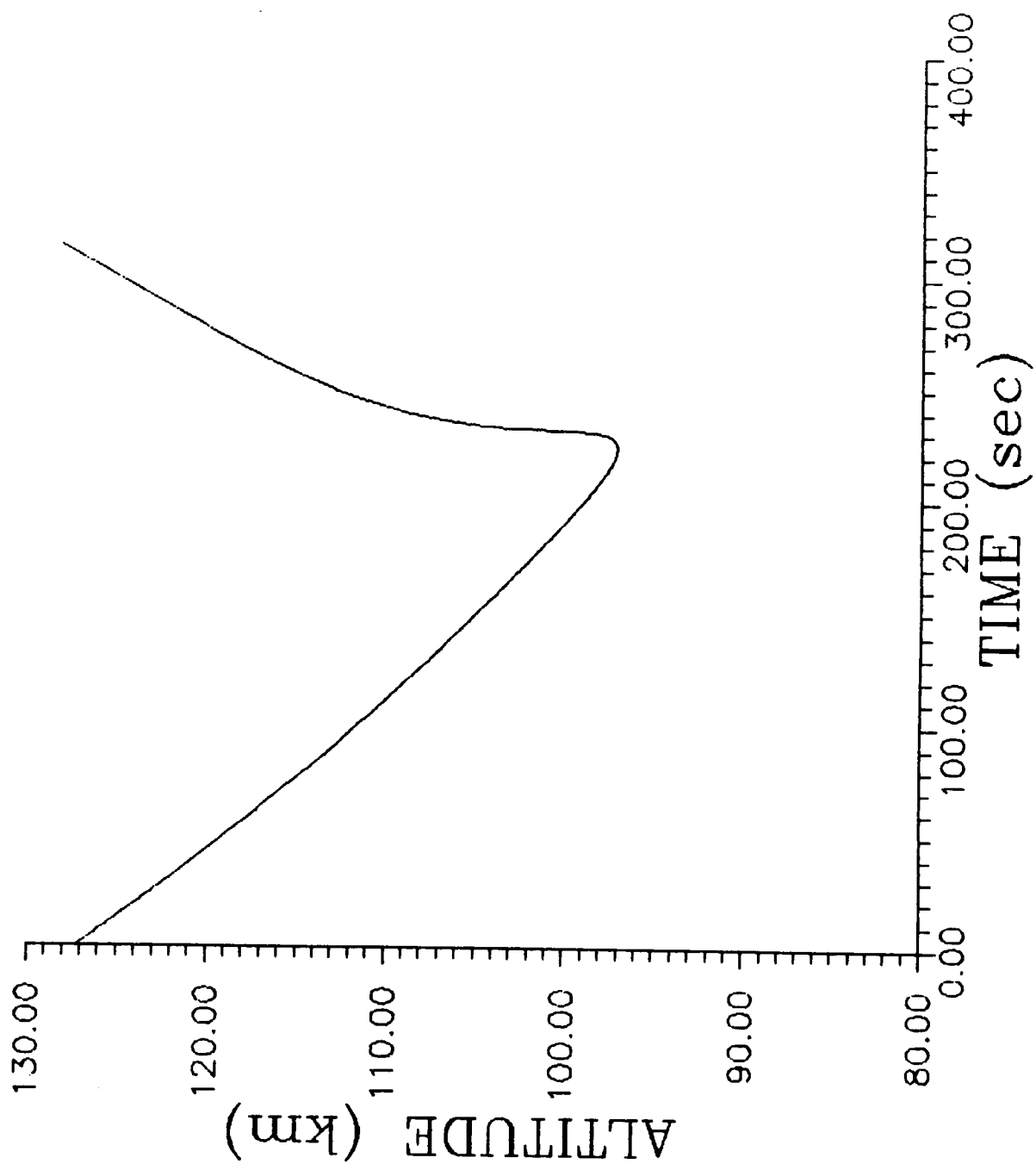
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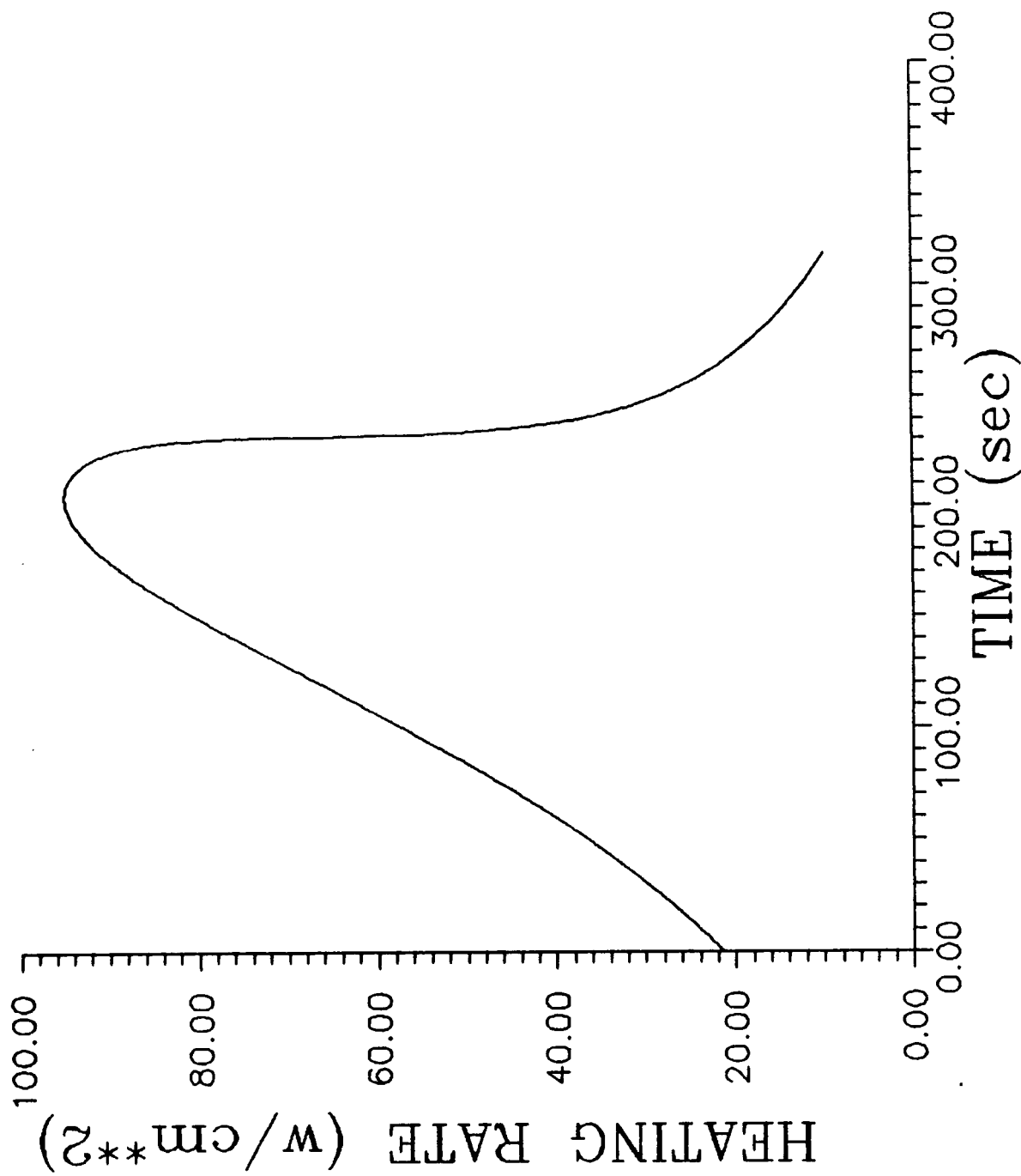
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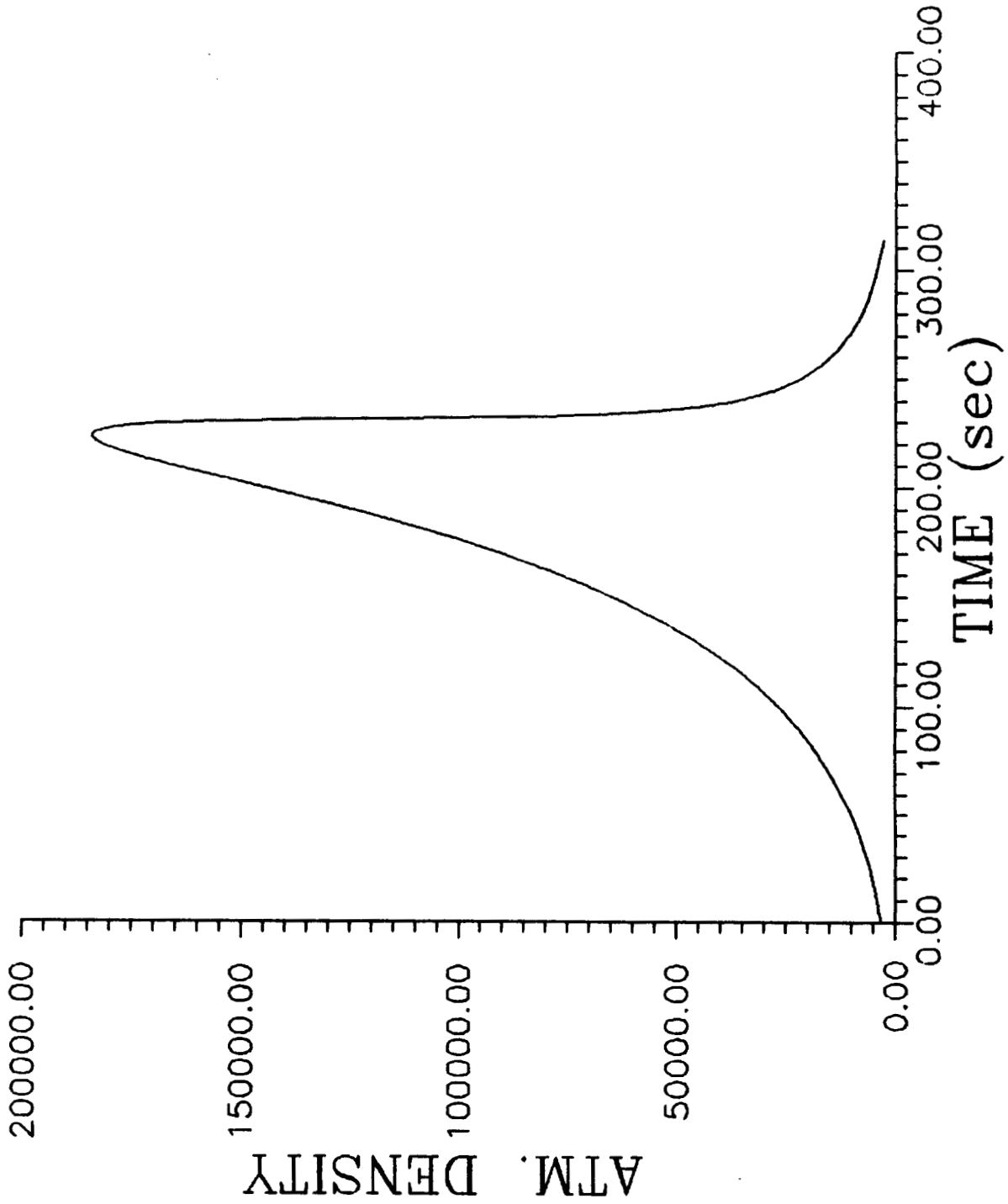
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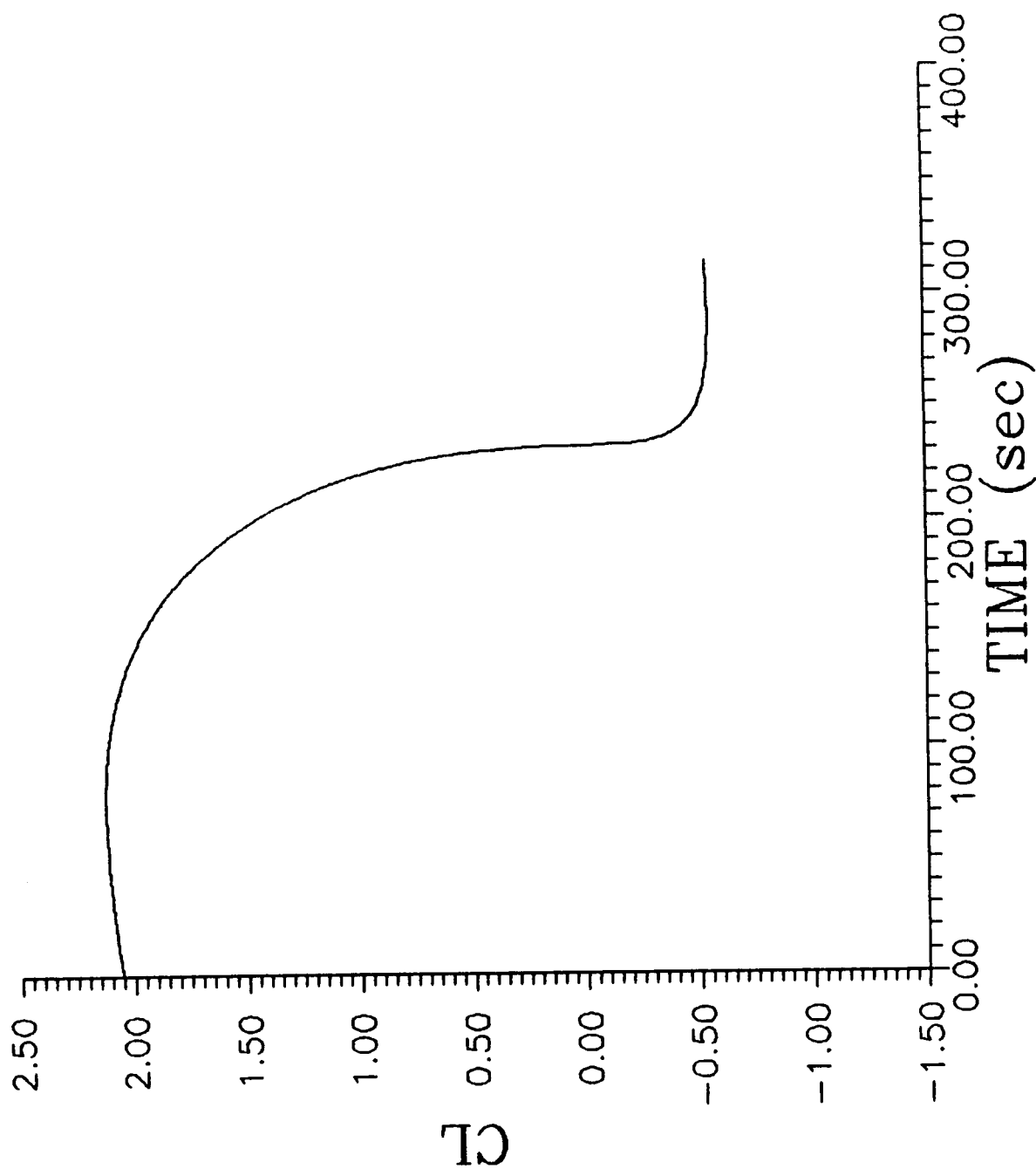
Figure 3.f. Optimal Guidance Trajectory for $\bar{\varepsilon}_1 = .001, \bar{\varepsilon}_2 = 1.0$

Figure 3.g. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 1.0$

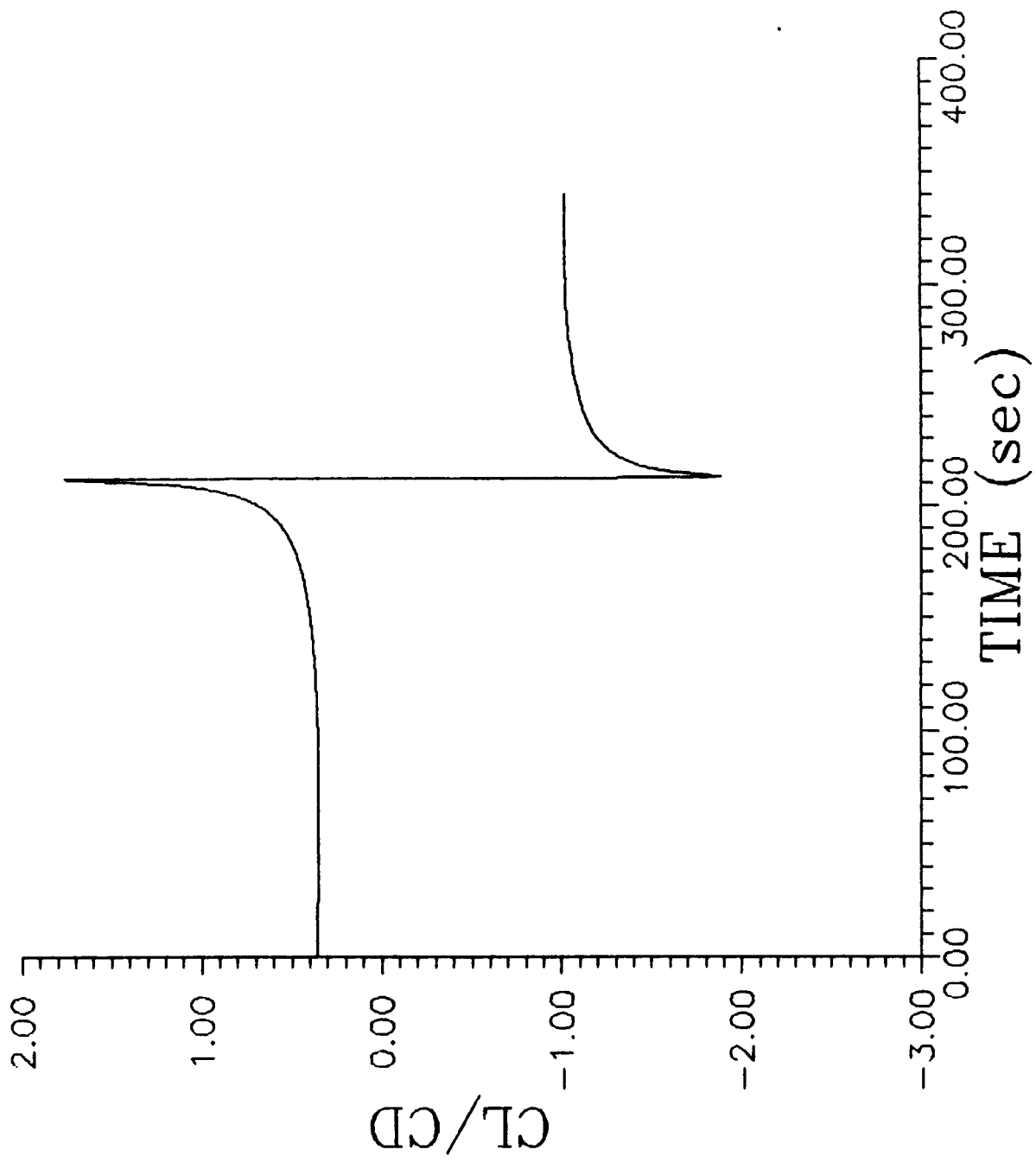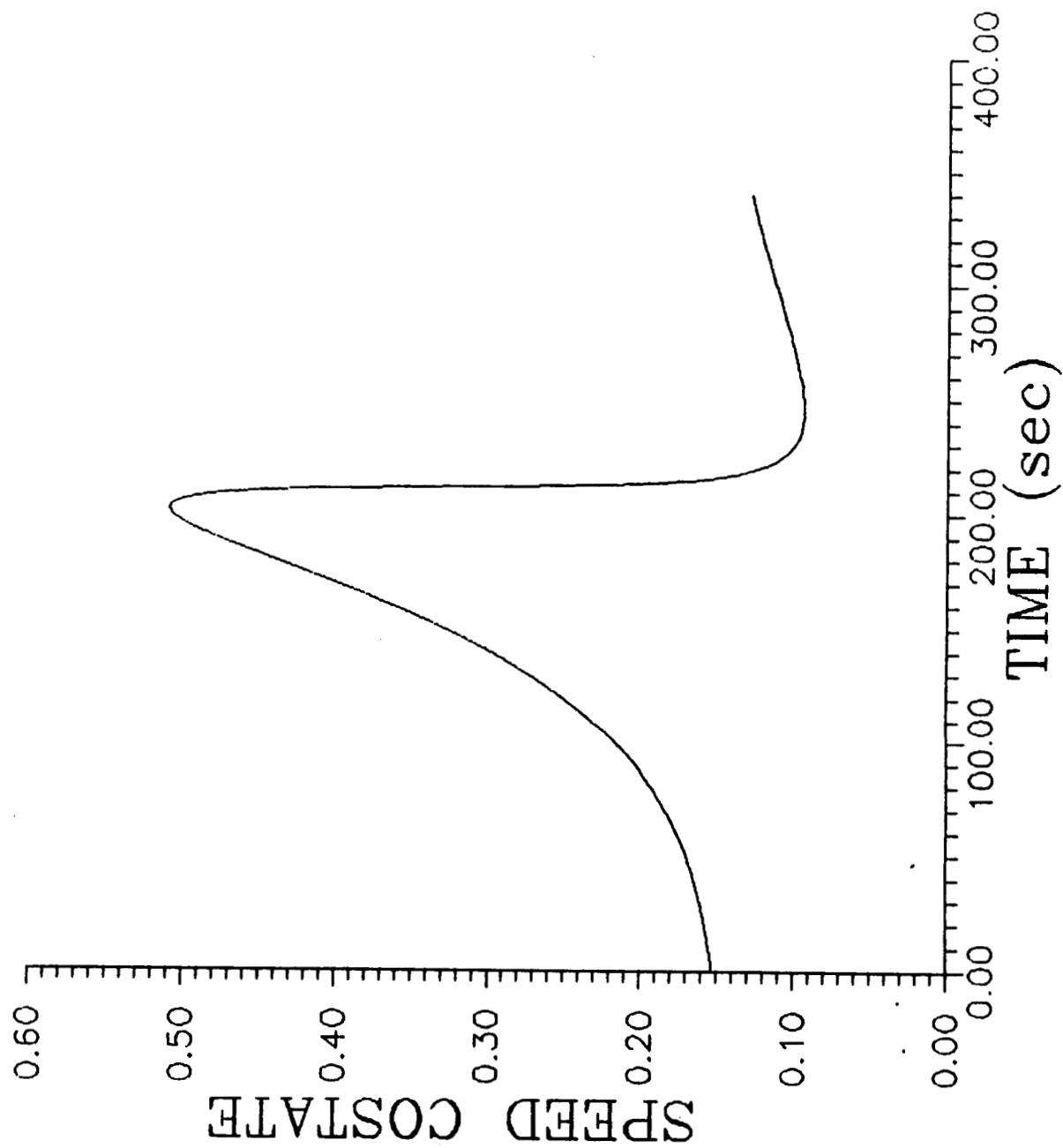Figure 3.h. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 1.0$

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Figure 3.i. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 1.0$

41

Figure 3.j. Optimal Guidance Trajectory for $\bar{\varepsilon}_1 = .001, \bar{\varepsilon}_2 = 1.0$

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Figure 4.a. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

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Figure 4.b. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

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Figure 4.c. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

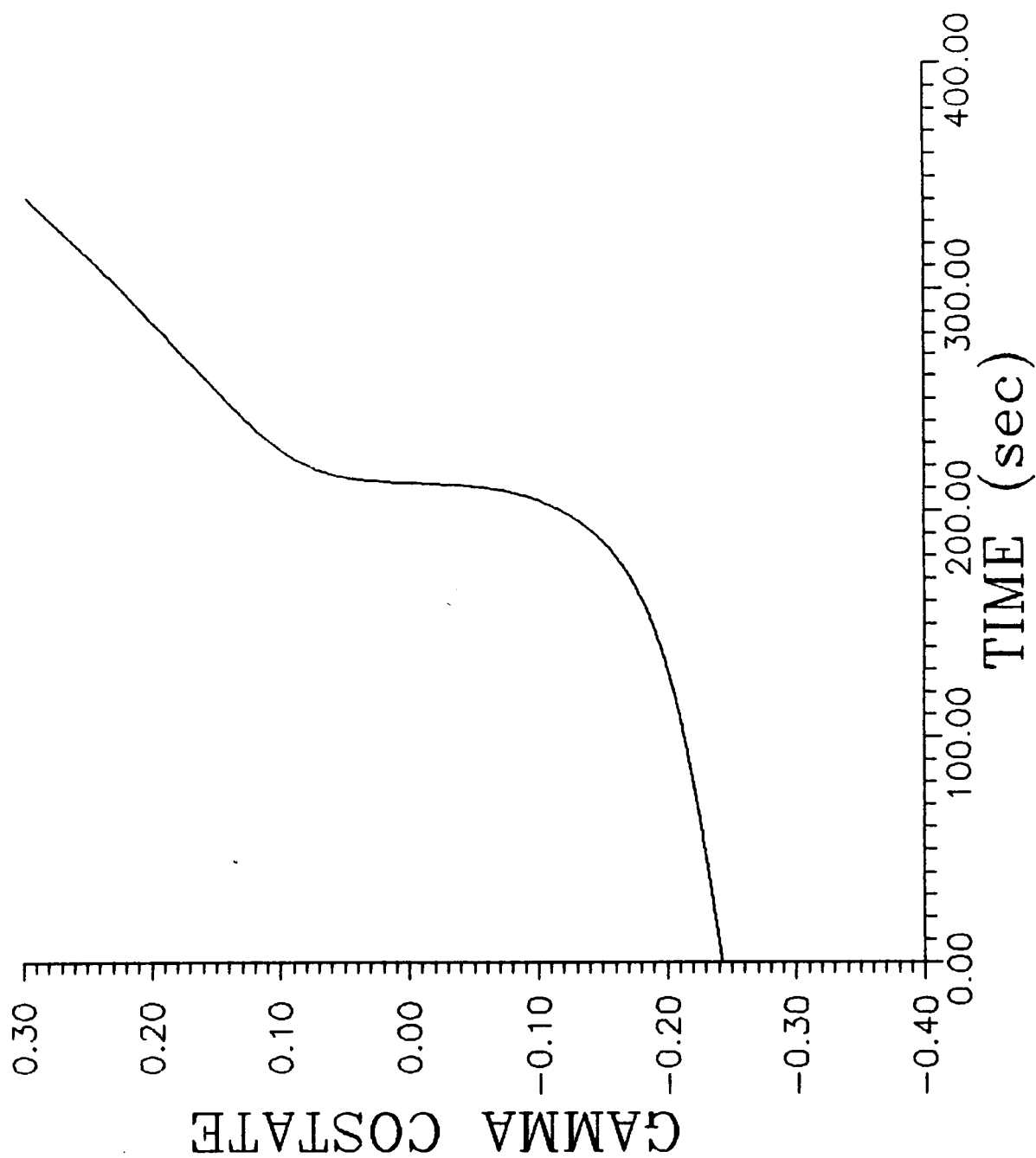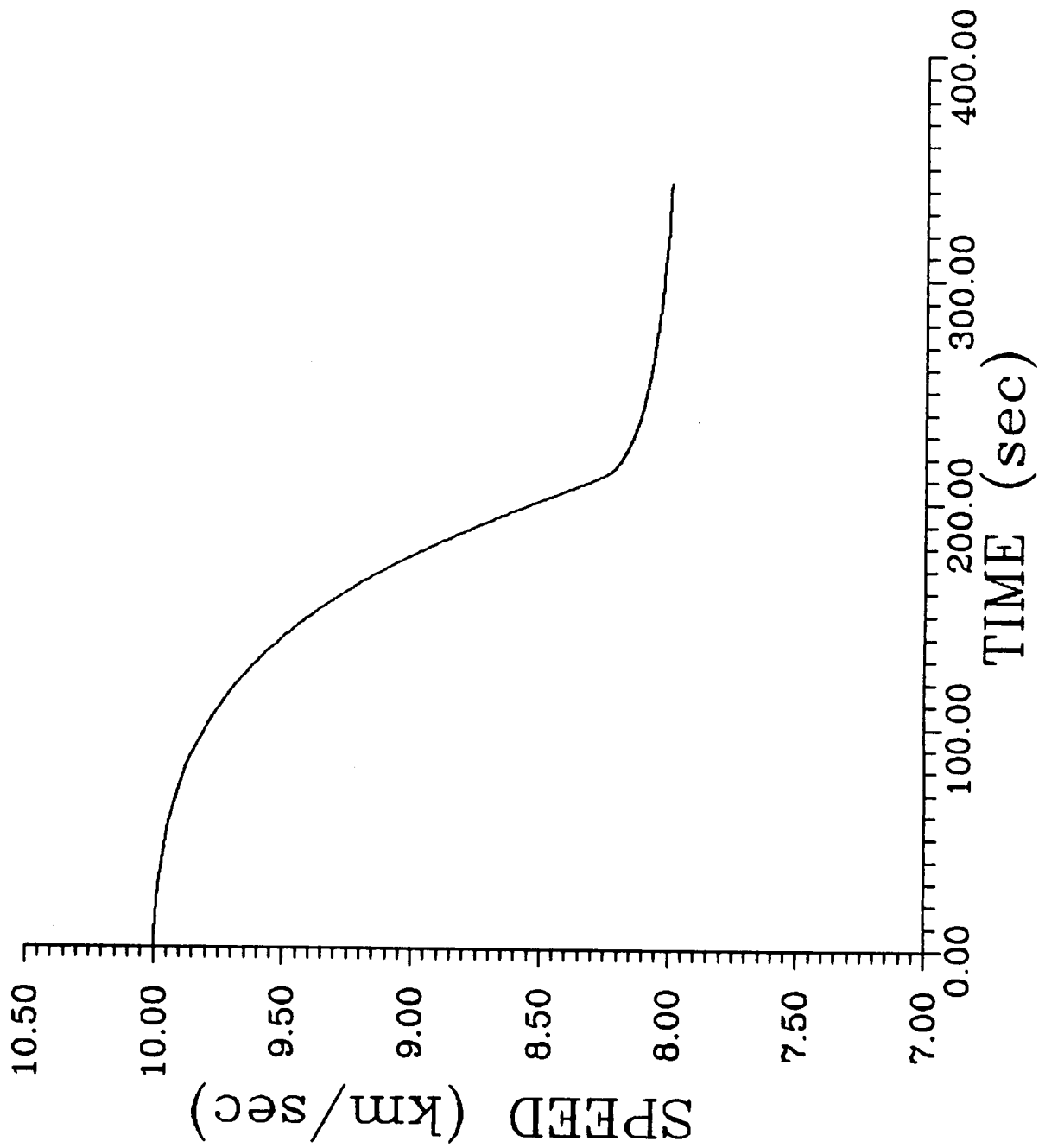Figure 4.d. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$
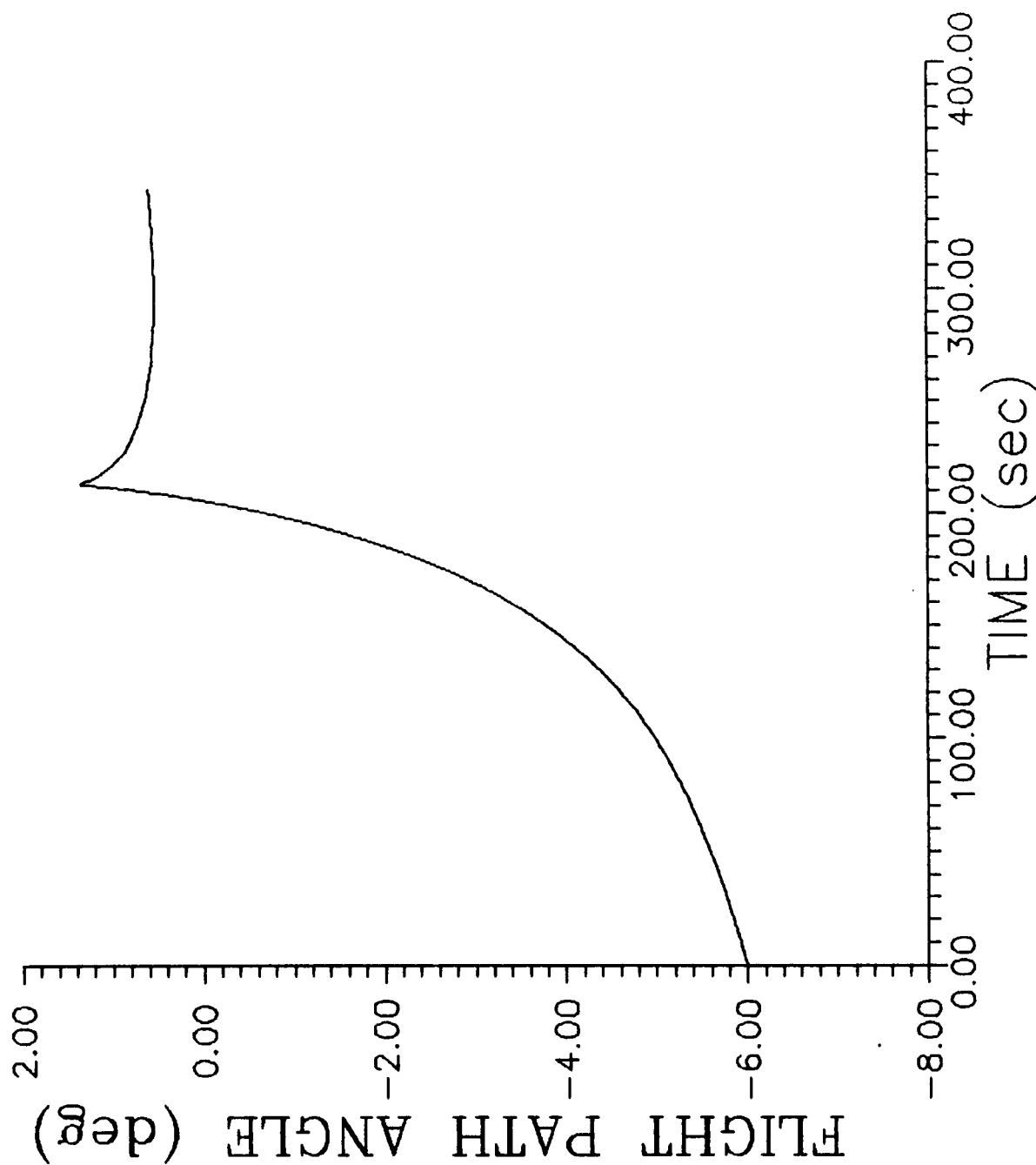
TIME (sec)

HEATING RATE (w/cm**2)

Figure 4.e. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

Figure 4.f. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

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Figure 4.g. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

Figure 4.h. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

Figure 4.i. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

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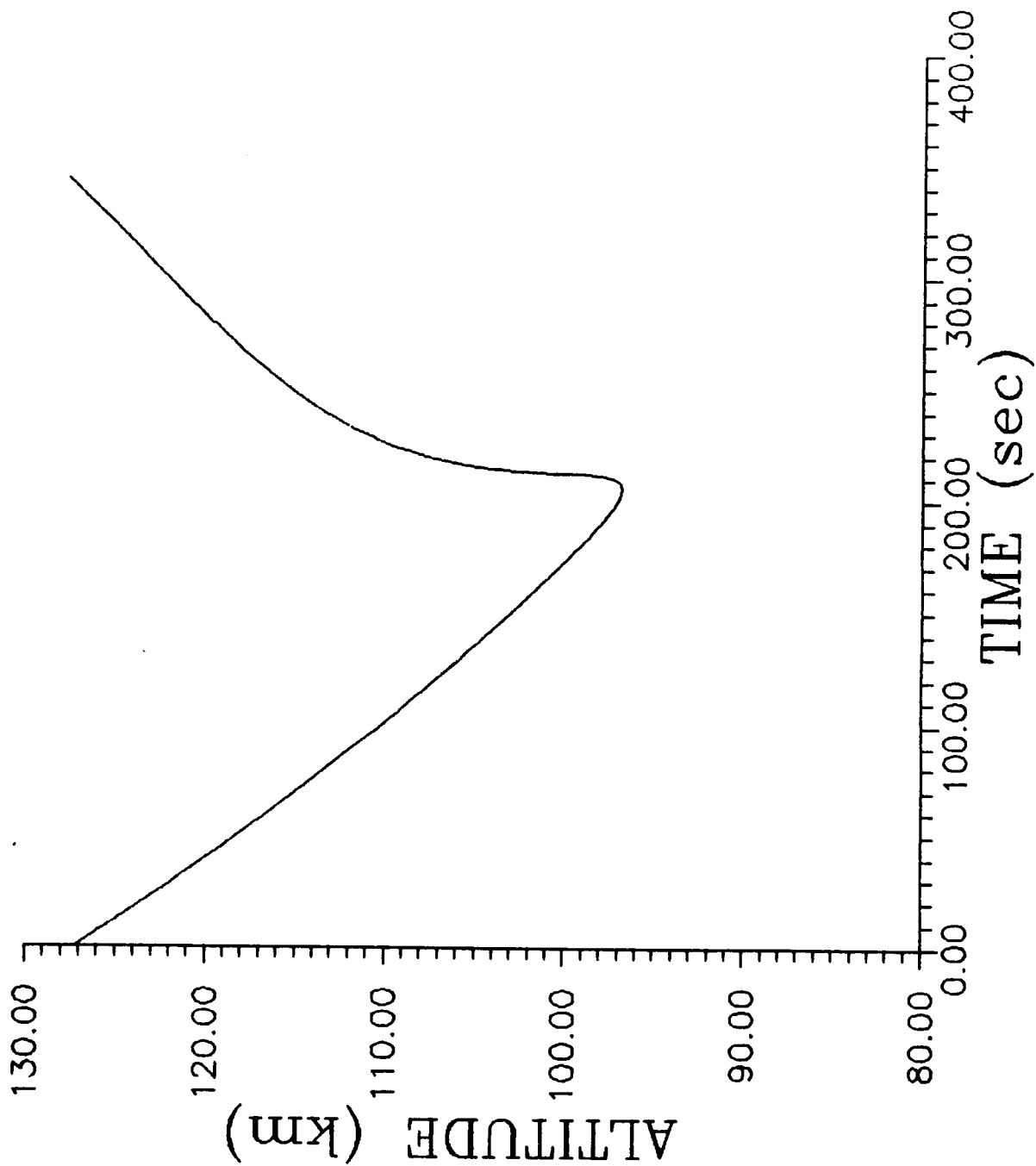Figure 4.j. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.6$

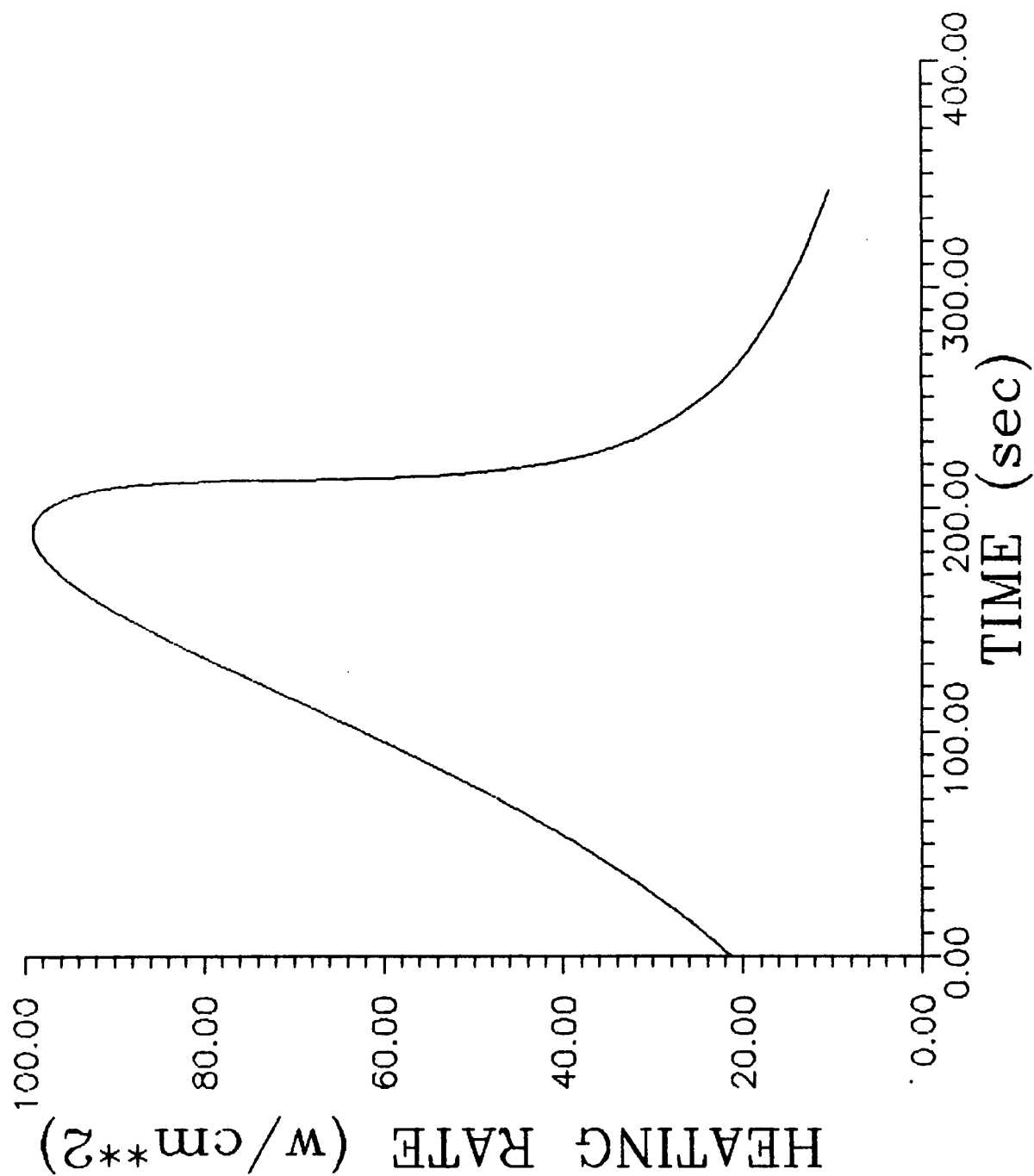Figure 5.a. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

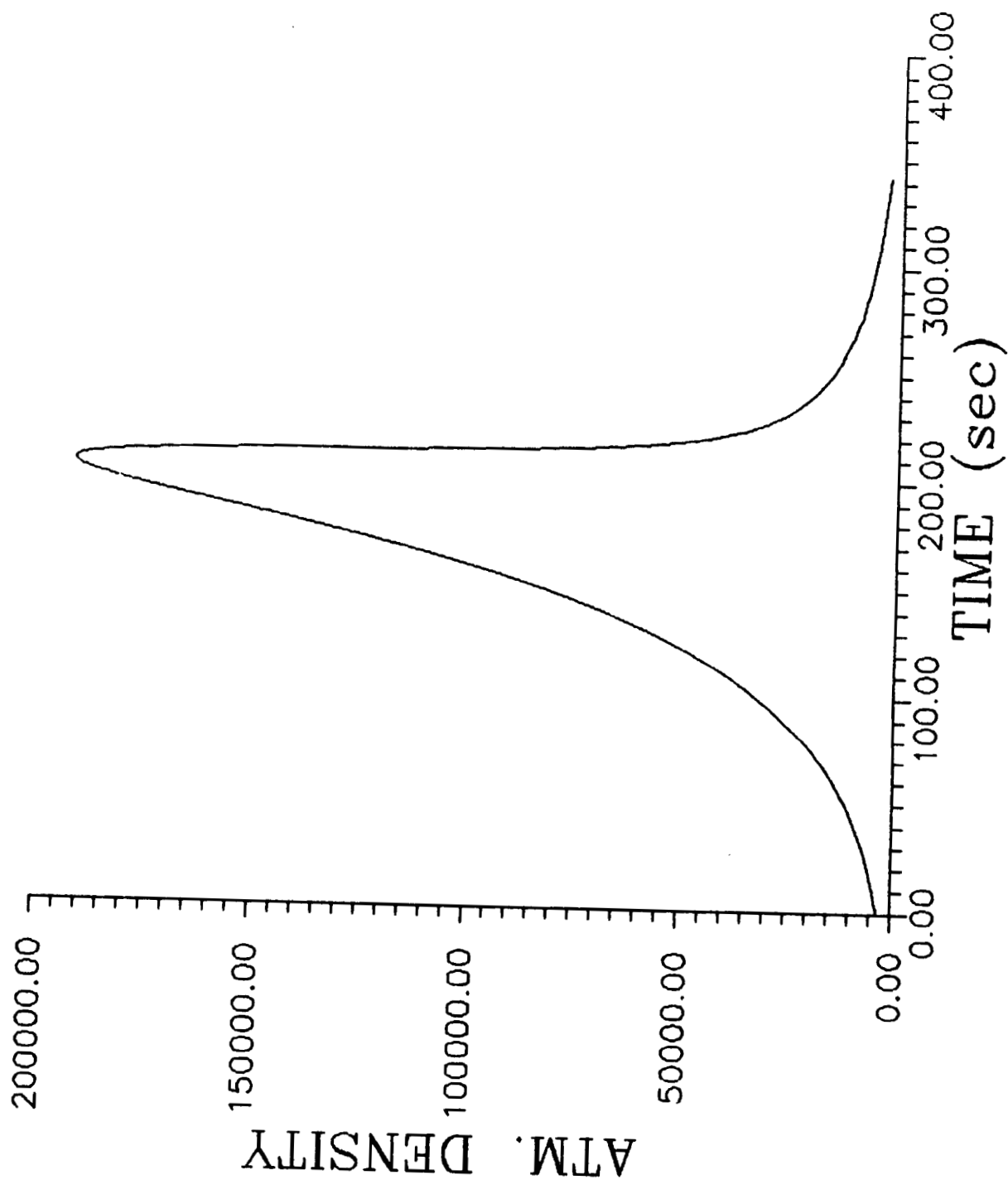Figure 5.b. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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Figure 5.c. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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Figure 5.d. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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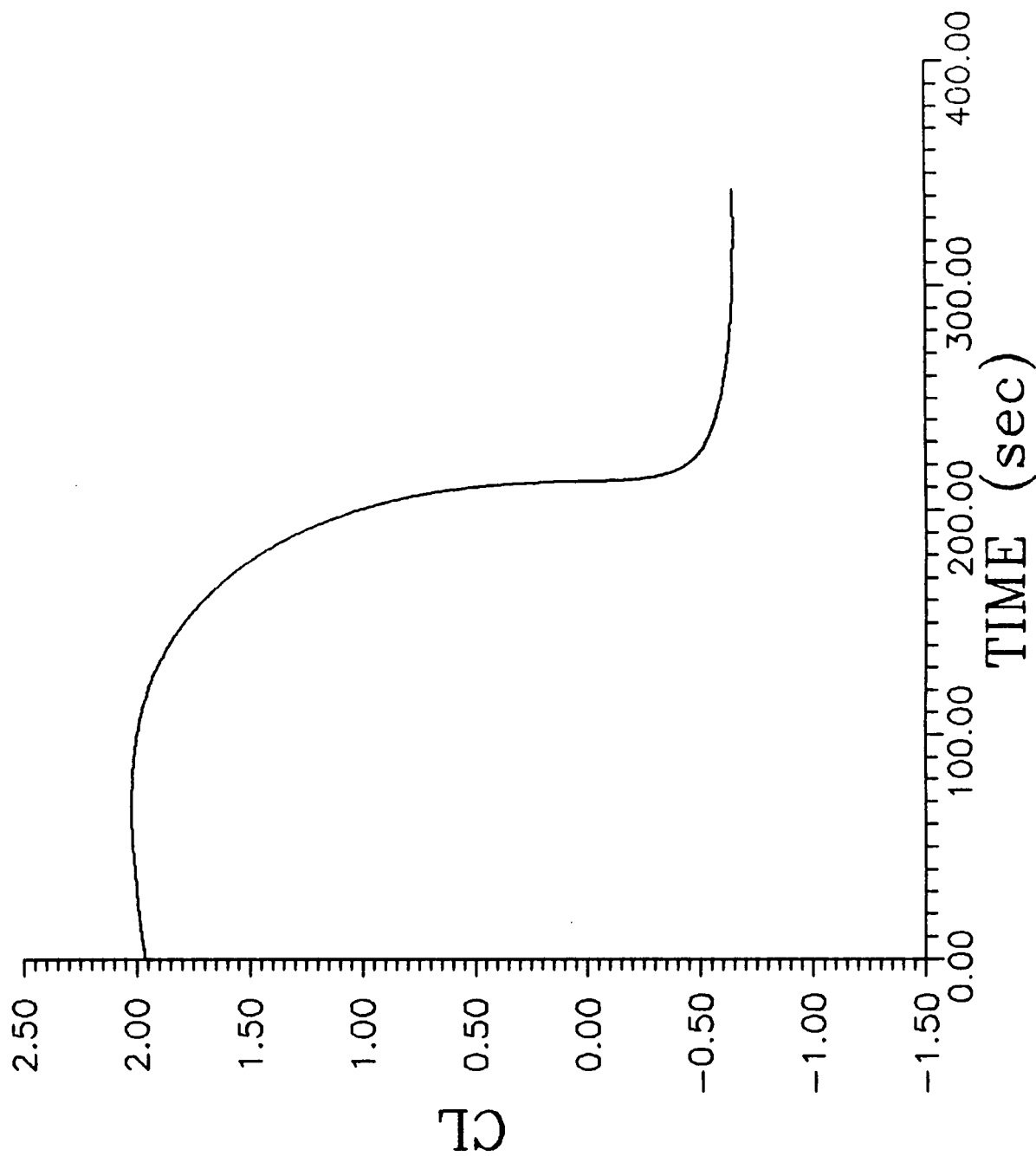Figure 5.e. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

Figure 5.f. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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Figure 5.g. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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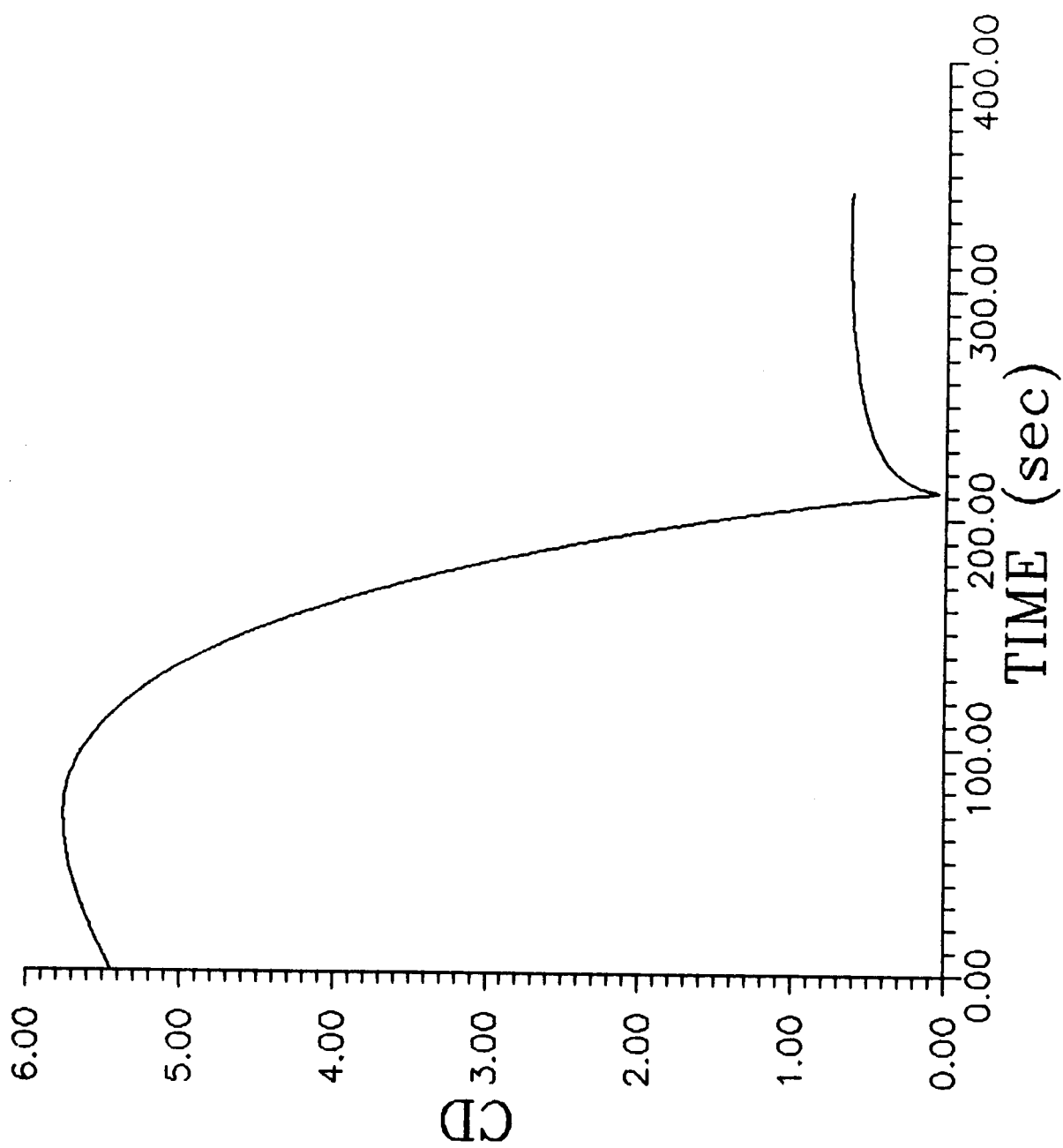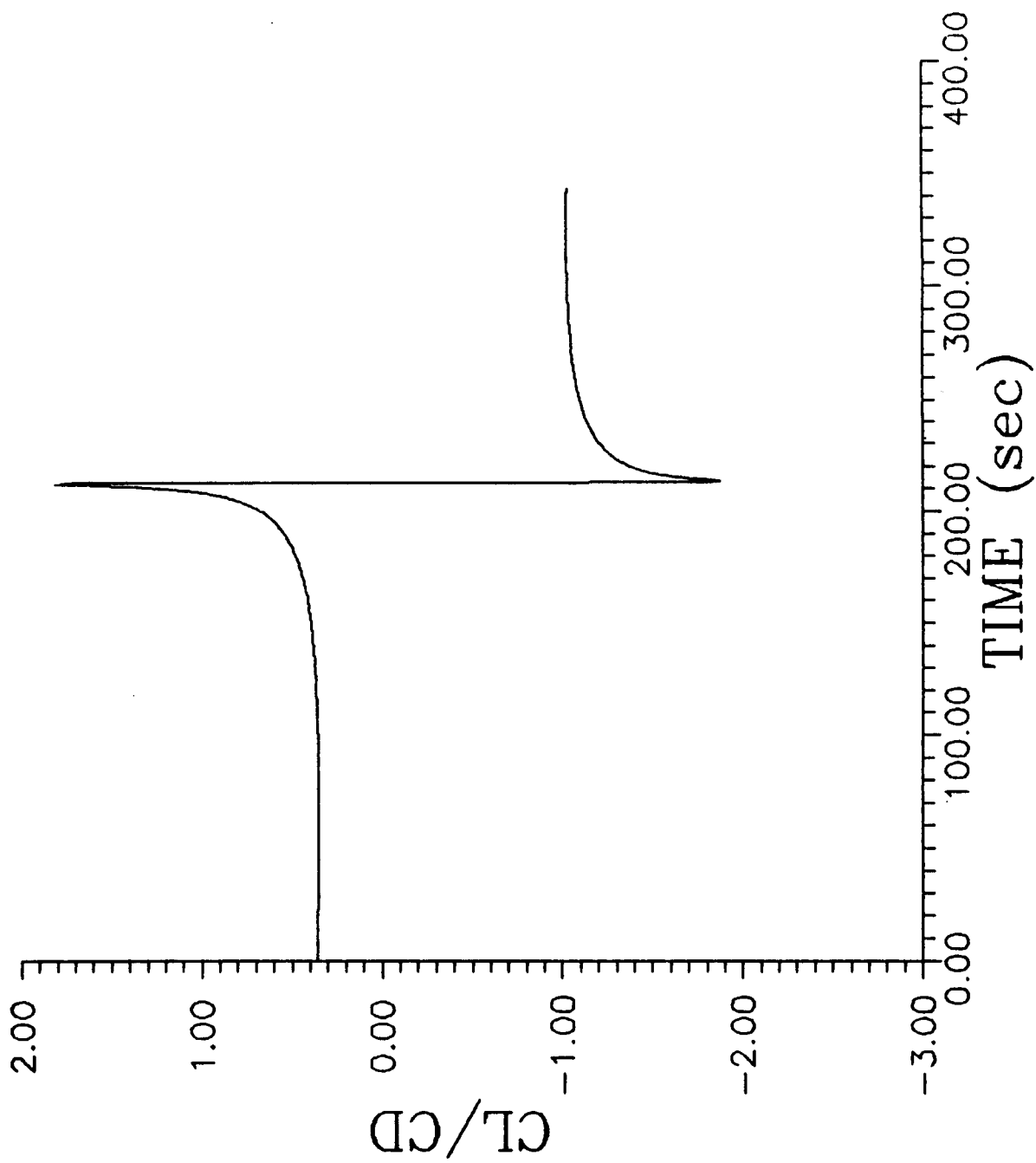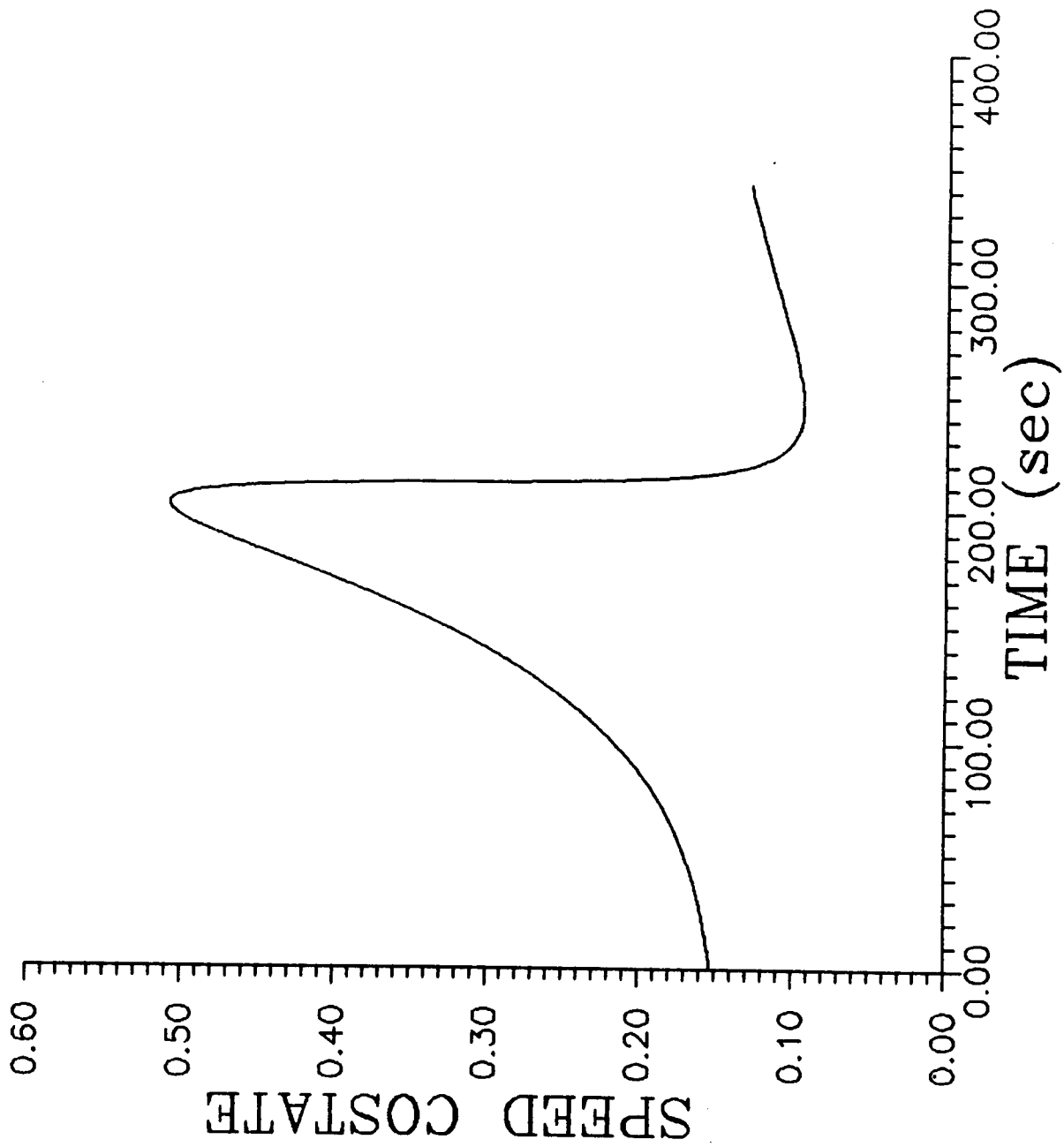
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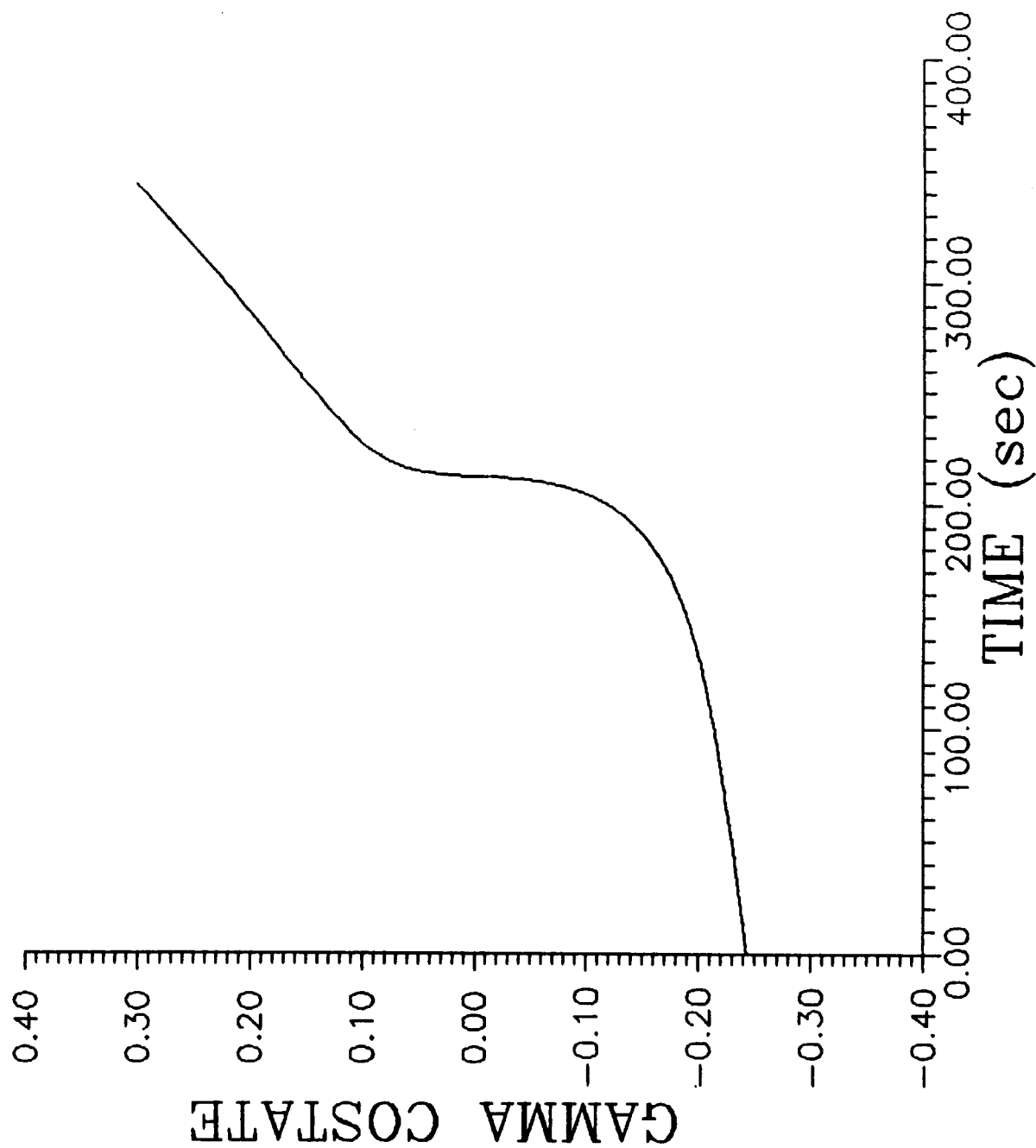Figure 5.j. Optimal Guidance Trajectory for $\bar{c}_1 = .001, \bar{c}_2 = 0.0$

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Report Documentation Page

1. Report No. NASA CR-181778	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle OPTIMAL AEROASSISTED COPLANAR ORBITAL TRANSFER USING AN ENERGY MODEL		5. Report Date May 1989
		6. Performing Organization Code
7. Author(s) NESIM HALYO DEBORAH B. TAYLOR		8. Performing Organization Report No. FR-688107
		10. Work Unit No.
9. Performing Organization Name and Address Information & Control Systems, Incorporated 28 Research Drive Hampton, VA 23666		11. Contract or Grant No. NAS1-17493
		13. Type of Report and Period Covered Final Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC		
		14. Sponsoring Agency Code

15. Supplementary Notes

Richard M. Hueschen, Technical Representative

16. Abstract

The atmospheric portion of the trajectories for the aeroassisted coplanar orbit transfer have been investigated. The equations of motion for the problem are expressed using a new reduced order model using total vehicle energy, kinetic plus potential, as the independent variable rather than time. The order reduction is achieved analytically without an approximation of the vehicle dynamics.

In this model, the problem of coplanar orbit transfer is seen as one in which a given amount of energy must be transferred from the vehicle to the atmosphere during the trajectory without overheating the vehicle. An optimal control problem is posed where a linear combination of the integrated square of the heating rate and the vehicle drag is the cost function to be minimized. The necessary conditions for optimality are obtained. These result in a 4th order two-point-boundary-value problem.

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17. Key Words (Suggested by Author(s)) aeroassisted orbital transfer, optimal guidance, energy model, reduced order model, coplanar orbital transfer	18. Distribution Statement		
19. Security Classif. (of this report) unclassified	20. Security Classif. (of this page) unclassified	21. No. of pages 65	22. Price

NASA FORM 1626 OCT 86